Problem Set 8

Problem 8.1 (Realizations of repetition and SPC codes)
Show that a reduced Hadamard transform realization of a repetition code RM(0, m) or a single-parity-check code RM(m − 1, m) is a cycle-free tree-structured realization with a minimum number of (3, 1, 3) repetition constraints or (3, 2, 2) parity-check constraints, respectively, and furthermore with minimum diameter (distance between any two code symbols in the tree). Show that these two realizations are duals; i.e., one is obtained from the other via interchange of (3, 2, 2) constraints and (3, 1, 3) constraints.

Problem 8.2 (Dual realizations of RM codes)
Show that in general a Hadamard transform (HT) realization of any Reed-Muller code RM(r, m) is the dual of the HT realization of the dual code RM(m − r − 1, m); i.e., one is obtained from the other via interchange of (3, 2, 2) constraints and (3, 1, 3) constraints.

Problem 8.3 (BCJR (sum-product) decoding of SPC codes)
As shown in Problem 6.4, any (μ+1, μ, 2) binary linear SPC block code may be represented by a two-state trellis diagram. Let μ = 7, and let the received sequence from a discrete-time AWGN channel be given by \( r = (0.1, -1.0, -0.7, 0.8, 1.1, 0.3, -0.9, 0.5) \). Perform BCJR (sum-product) decoding of this sequence, using the two-state trellis diagram of the (8, 7, 2) SPC code.

Compare the performance and complexity of BCJR decoding to that of the Viterbi algorithm and Wagner decoding (Problem 6.6).