Announcements
- Pick up random processes notes, lecture notes, slides

Continuous-Time Photodetection
- Semiclassical theory — Poisson shot noise
- Quantum theory — photon-flux operator measurement
- Direct-detection signatures of non-classical light
Real Photodetection Systems

“Ideal” Photodetection System: Efficiency < 1
Classical Field versus Quantum Field Operator

- For Semiclassical Photodetection of Narrowband Light
  - Illumination is a classical photon-units positive-frequency field:
    \[ E(t)e^{-j\omega_0 t} \]
  - Short-Time Average Power on Detector:
    \[ P(t) = \hbar\omega_0 |E(t)|^2 \]

- For Quantum Photodetection of Narrowband Light
  - Illumination is a photon-units positive-frequency field operator:
    \[ \hat{E}'(t)e^{-j\omega_0 t} \]
  - Only non-vacuum frequency components are within \( \Delta\omega \ll \omega_0 \)

Semiclassical versus Quantum Photodetection

- Semiclassical Theory: Given \( \{ P(t) : 0 \leq t \leq T \} \)
  - \( \{ N(t) : 0 \leq t \leq T \} \) is an inhomogeneous Poisson Counting Process
  - Rate function \( \lambda(t) \equiv \eta P(t)/\hbar\omega_0 \)

- Quantum Theory:
  \[
  \hat{E}'(t) \equiv \sqrt{\eta} \hat{E}(t) + \sqrt{1-\eta} \hat{E}_\eta(t) \\
  \hat{i}(t) \leftrightarrow \hat{i}(t) \equiv q\hat{E}'(t)\hat{E}'(t) \\
  N(t) \leftrightarrow \frac{1}{\eta} \int_0^t d\tau \hat{i}(\tau), \quad \text{for } 0 \leq t \leq T
  \]
Inhomogeneous Poisson Counting Process (IPCP)

- Definition: \( \{ N(t) : t \geq 0 \} \) is an IPCP with rate \( \lambda(t) \)
  - Starts counting at zero: \( N(0) = 0 \)
  - Has statistically independent increments
  - Increments are Poisson distributed

\[
Pr(N(t) - N(u) = n) = \frac{\left( \int_u^t d\tau \lambda(\tau) \right)^n}{n!} \exp\left(- \int_u^t d\tau \lambda(\tau)\right)
\]

Mean and Covariance: Deterministic Illumination

- Semiclassical Photocount and Photocurrent Mean Functions:
  \[
  \langle N(t) \rangle = \int_0^t d\tau \frac{\eta P(\tau)}{\hbar \omega_o} \quad \text{and} \quad \langle i(t) \rangle = q \frac{\eta P(t)}{\hbar \omega_o}
  \]

- Semiclassical Covariance Functions:
  \[
  \langle \Delta N(t) \Delta N(u) \rangle = \langle N(\min(t, u)) \rangle
  \]
  \[
  \langle \Delta i(t) \Delta i(u) \rangle = q \langle i(t) \rangle \delta(t-u)
  \]
Mean and Covariance: Random Illumination

- Semiclassical Photocount and Photocurrent Mean Functions:
  \[
  \langle N(t) \rangle = \int_0^t d\tau \frac{\eta \langle P(\tau) \rangle}{\hbar \omega_o}
  \]
  \[
  \langle i(t) \rangle = q \frac{\eta \langle P(\tau) \rangle}{\hbar \omega_o}
  \]

- Semiclassical Covariance Functions:
  \[
  \langle \Delta N(t) \Delta N(u) \rangle = \langle N(\min(t,u)) \rangle + \int_0^t d\tau \int_0^u d\tau' \eta^2 \frac{\langle \Delta P(\tau) \Delta P(\tau') \rangle}{(\hbar \omega_o)^2}
  \]
  \[
  \langle \Delta i(t) \Delta i(u) \rangle = q \langle i(t) \rangle \delta(t-u) + q^2 \eta^2 \frac{\langle \Delta P(t) \Delta P(u) \rangle}{(\hbar \omega_o)^2}
  \]

Mean and Covariance Functions: Quantum Case

- Quantum Photocount and Photocurrent Mean Functions:
  \[
  \langle N(t) \rangle = \int_0^t d\tau \eta \langle \hat{E}^\dagger(\tau) \hat{E}(\tau) \rangle \quad \text{and} \quad \langle i(t) \rangle = q \eta \langle \hat{E}^\dagger(t) \hat{E}(t) \rangle
  \]

- Quantum Covariance Functions:
  \[
  \langle \Delta N(t) \Delta N(u) \rangle = \langle N(\min(t,u)) \rangle
  \]
  \[
  + \int_0^t d\tau \int_0^u d\tau' \eta^2 \left[ \langle \hat{E}^\dagger(\tau) \hat{E}^\dagger(\tau') \hat{E}(\tau) \hat{E}(\tau') \rangle - \langle \hat{E}^\dagger(\tau) \hat{E}(\tau) \rangle \langle \hat{E}^\dagger(\tau') \hat{E}(\tau') \rangle \right]
  \]
  \[
  \langle \Delta i(t) \Delta i(u) \rangle = q \langle i(t) \rangle \delta(t-u)
  \]
  \[
  + q^2 \eta^2 \left[ \langle \hat{E}^\dagger(t) \hat{E}^\dagger(u) \hat{E}(t) \hat{E}(u) \rangle - \langle \hat{E}^\dagger(t) \hat{E}(t) \rangle \langle \hat{E}^\dagger(u) \hat{E}(u) \rangle \right]
  \]
Direct-Detection Signatures of Non-Classical Light

- Semiclassical Theory is Quantitatively Correct:
  - For coherent-state inputs $|E(t)\rangle$
  - For inputs that are classically-random mixtures of coherent states

- Sub-Poissonian Photon Counting:
  - Semiclassical theory:
    \[
    \langle \Delta N^2(t) \rangle \geq \langle N(t) \rangle
    \]
  - Quantum theory:
    \[
    \langle \Delta N^2(t) \rangle \geq 0
    \]
  - Non-classical signature:
    \[
    \langle \Delta N^2(t) \rangle < \langle N(t) \rangle
    \]

Direct-Detection Signatures of Non-Classical Light

- Photocurrent Noise Spectral Density for CW Sources:
  \[
  S_{ii}(\omega) \equiv \int_{-\infty}^{\infty} d\tau \langle \Delta i(t+\tau)\Delta i(t) \rangle e^{-j\omega \tau}
  \]
  - Semiclassical Theory:
    \[
    S_{ii}(\omega) = q\langle i \rangle + q^2 \eta^2 S_{PP}(\omega) \left(\frac{1}{\hbar \omega_o}\right)^2 \geq q\langle i \rangle
    \]
  - Quantum Theory:
    \[
    S_{ii}(\omega) \geq 0
    \]
  - Sub-Shot-Noise Non-Classical Signature:
    \[
    S_{ii}(\omega) < q\langle i \rangle
    \]
Coming Attractions: Lecture 19

- Lecture 19: Continuous-Time Photodetection
  - Noise spectral densities in direct detection
  - Semiclassical theory of coherent detection
  - Quantum theory of coherent detection
  - Coherent-detection signatures of non-classical light