Announcements
- Pick up lecture notes, slides
- Term papers are due Tuesday, December 13th

More Quantum Optical Applications
- Binary optical communication with squeezed states
- Phase-sensing interferometry with squeezed states
- Super-dense coding with entangled states
- Quantum lithography with “N00N” states
Minimum Probability of Error Binary Communication

- Binary Phase-Shift-Keying with Coherent States

\[ m = 0 \text{ or } 1 \quad |\psi_m\rangle = (-1)^{m+1}\sqrt{N} \]

\[ \Delta \rho \]

\[ \tilde{m} = 0 \text{ or } 1 \]

- Optimum Decision Rule and Minimum Probability of Error

\[ \tilde{m} = 1 \]

\[ \Delta \rho \geq 0, \quad \text{Pr}(e) \approx \frac{1}{4}e^{-4N}, \text{ for } N \gg 1 \]

Minimum Probability of Error Binary Communication

- Binary Phase-Shift-Keying with Squeezed States

\[ m = 0 \text{ or } 1 \quad |\psi_m\rangle = (-1)^{m+1}\beta; \mu, \nu \]

\[ \Delta \rho \]

\[ \tilde{m} = 0 \text{ or } 1 \]

- Optimum Decision Rule and Minimum Probability of Error

\[ \tilde{m} = 1 \]

\[ \Delta \rho \geq 0, \quad \text{Pr}(e) \approx \frac{1}{4}e^{-4N^2}, \text{ for } N \gg 1 \]
Phase-Sensing Interferometry with Coherent States

- Phase-Conjugate Mach-Zehnder Interferometer: \(|\phi| \ll 1\)
  
  \[ \begin{array}{c}
  \hat{b}_{\text{in}} \\
  \downarrow \\
  \hat{a}_{\text{in}} \\
  |\sqrt{N}\rangle \\
  \downarrow \\
  \hat{b}_{\text{out}} \\
  \downarrow \\
  \hat{a}_{\text{out}} \\
  \phi \\
  \end{array} \]

- Homodyne Measurement of \(\tilde{\phi} \leftrightarrow -\text{Im}(\hat{b}_{\text{out}})/\sqrt{N}\)
  \[
  \langle \tilde{\phi} \rangle = \phi \quad \langle \Delta \tilde{\phi}^2 \rangle = 1/4N
  \]

Phase-Sensing Interferometry with Squeezed States

- Phase-Conjugate Mach-Zehnder Interferometer: \(|\phi| \ll 1\)
  
  \[ \begin{array}{c}
  \hat{b}_{\text{in}} \\
  \downarrow \\
  \hat{a}_{\text{in}} \\
  |\beta; \mu, \nu\rangle \\
  \downarrow \\
  \hat{b}_{\text{out}} \\
  \downarrow \\
  \hat{a}_{\text{out}} \\
  \phi \\
  \end{array} \]

- Homodyne Measurement of \(\tilde{\phi} \leftrightarrow -\text{Im}(\hat{b}_{\text{out}})/\sqrt{N - 2\nu^2}\)
  \[
  \langle \tilde{\phi} \rangle = \phi \quad \langle \Delta \tilde{\phi}^2 \rangle = 1/2N(N + 2)
  \]
Binary Communication with Single Photons

- Binary Polarization Modulation, Lossless Channel, $\eta = 1$

*message source*

$m = 0 \text{ or } 1$

1-photon source

*$\psi_m$

$|\psi_0\rangle = |H\rangle$

$|\psi_1\rangle = |V\rangle$

PBS

$\tilde{m} = 1$ if click

$\tilde{m} = 0$ if click

- One Bit of Information Transmitted per Photon

Super-Dense Coding with Entangled Photons

- Alice and Bob Share a Singlet State of Two Photons:

  \[
  |\psi^-\rangle_{AB} = \frac{|H\rangle_A |V\rangle_B - |V\rangle_A |H\rangle_B}{\sqrt{2}}
  \]

- Alice Uses Two Classical Bits to Modulate Her Photon:

  \[
  \alpha |H\rangle_A + \beta |V\rangle_A \quad \longrightarrow \quad \alpha |H\rangle_A + \beta |V\rangle_A, \quad \text{if } m = 00
  \]

  \[
  \alpha |H\rangle_A + \beta |V\rangle_A \quad \longrightarrow \quad \alpha |H\rangle_A - \beta |V\rangle_A, \quad \text{if } m = 01
  \]

  \[
  \alpha |H\rangle_A + \beta |V\rangle_A \quad \longrightarrow \quad \alpha |V\rangle_A + \beta |H\rangle_A, \quad \text{if } m = 10
  \]

  \[
  \alpha |H\rangle_A + \beta |V\rangle_A \quad \longrightarrow \quad \alpha |V\rangle_A - \beta |H\rangle_A, \quad \text{if } m = 11
  \]
Super-Dense Coding with Entangled Photons

- Alice Sends Her Photon to Bob
  - Bob then has a Bell state:
    \[ \frac{|H\rangle_A |V\rangle_B - |V\rangle_A |H\rangle_B}{\sqrt{2}} \text{, if } m = 00 \]
    \[ \frac{|H\rangle_A |V\rangle_B + |V\rangle_A |H\rangle_B}{\sqrt{2}} \text{, if } m = 01 \]
    \[ \frac{|H\rangle_A |H\rangle_B - |V\rangle_A |V\rangle_B}{\sqrt{2}} \text{, if } m = 10 \]
    \[ \frac{|H\rangle_A |H\rangle_B + |V\rangle_A |V\rangle_B}{\sqrt{2}} \text{, if } m = 11 \]
- Bob Makes the Bell-Observable Measurement
  - Bob decodes both of Alice’s bits without error

Optical Lithography with Coherent States

- Interference Between Plane Waves on a Photoresist
  \[ \frac{\hat{a}_{\pm} e^{-j(\omega t - \vec{k}_\pm \cdot \vec{r})}}{\sqrt{T}} |\sqrt{N/2}\rangle \]
  \[ \frac{\hat{a}_{\pm} e^{-j(\omega t - \vec{k}_\mp \cdot \vec{r})}}{\sqrt{T}} |\sqrt{N/2}\rangle \]

\[ \vec{k}_\pm = \pm k \sin(\theta) \hat{r}_x + k \cos(\theta) \hat{r}_z \]

\[ \left\langle \int_0^T dt \, \hat{E}^\dagger(x, t) \hat{E}(x, t) \right\rangle = N [1 + \cos(2k \sin(\theta) x)] \]
**Optical Lithography with “N00N” States**

- Interference Between Plane Waves on an $N$-Photon Resist

\[
\hat{a}_+ e^{-j(\omega t - \vec{k}_+ \cdot \vec{r})} \sqrt{T} + \hat{a}_- e^{-j(\omega t - \vec{k}_- \cdot \vec{r})} \sqrt{T}
\]

\[
\frac{|N\rangle_+ |0\rangle_- + |0\rangle_+ |N\rangle_-}{\sqrt{2}}
\]

\[
\left\langle \int_0^T dt \, K \hat{E}^\dagger_N (x, t) \hat{E}^N (x, t) \right\rangle = K N! \left[ 1 + \cos(2kN \sin(\theta)x) \right] / T^{N-1}
\]

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**Subject Outline Revisited — We’re Done!**

- Quantum Optics
  - Dirac notation quantum mechanics; harmonic oscillator quantization; number states, coherent states, and squeezed states; $P$ representation and classical fields.
- Single-Mode and Two-Mode Quantum Systems
  - Direct, homodyne, and heterodyne detection; linear propagation loss; phase insensitive and phase sensitive amplifiers; entanglement and teleportation.
- Multi-Mode Quantum Systems
  - Field quantization; quantum photodetection.
- Nonlinear Optics
  - Phase-matched interactions; optical parametric amplifiers; generation of squeezed states, photon-twin beams, non-classical fourth-order interference, and polarization entanglement.
- Quantum Systems Theory
  - Optimum binary detection; quantum precision measurements; quantum cryptography.