Problem 1 (30%)

(a) A conducting bar with a current $I_1$ (with direction as indicated in Fig. 1(a)) is on a smooth slope. If there is an external static magnetic field $\vec{B}$ pointing upwards, as shown in Figure 1(a), write down the magnitude of $\vec{B}$ at which the bar can rest on the slope. (Assume the bar’s length is infinite and its gravity per unit length is $mg$.)

(b) Consider the setup as shown in Fig. 1(b). One conducting bar on the smooth slope is carrying a current $I_1$. Another conducting bar is carrying current $I_2$, sitting at the bottom of the slope, parallel to the former bar and keeping it at rest on the slope. What is $I_2$ and is $I_2$ flowing into the paper or out of the paper?

Problem 2 (30%)

Consider the diagram as shown in Fig. 2. An incident plane wave is propagating in a homogeneous medium with permittivity $\epsilon$ and permeability $\mu$ for $x < 0$. The magnetic field of the incident wave is

$$\vec{H}_1 = \hat{y} H_0 \cos\left(\frac{\sqrt{3}}{2} k z + \frac{1}{2} k x - \omega t\right)$$
(a) Write out the $\hat{k}$ vector and the electric field $E_1$ of the incident wave.
(b) Is the incident wave a TE or TM wave?
(c) The half space $x > 0$ is filled with another material with permittivity $3\epsilon$ and permeability $\mu$. What’s the reflection coefficient?
(d) Suppose the incident electric field is now $E = E_1 + E_2$. Find $E_2$ such that $E$ is a right-hand circularly polarized wave.
(e) If the wave is incident from the half space $x > 0$ instead of incident from the half space $x < 0$, what is the range of incident angles for which total reflection occurs?

Problem 3 (40%)

Two Hertzian dipole antennas are located at $(0, 0, 0)$ and $(0, 0, d)$ with dipole moments $p_1 = q_1l$ and $p_2 = q_2l$ as shown in Fig. 3. The two in-phase dipoles are oriented in $z$ and $x$ direction respectively.

(a) For the $x$-oriented dipole, the far field ($r \gg 1$) expression of electric field on the $yz$-plane is:

$$E_2 = \hat{x} \frac{k^2 q_2 \ell}{4\pi r \epsilon_0} \cos(k\sqrt{y^2 + (z - d)^2} - \omega t)$$

Show that as $d \ll \sqrt{y^2 + z^2} = r$,

$$E_2 = \hat{x} \frac{k^2 q_2 \ell}{4\pi r \epsilon_0} \cos(kr - kd \cos \theta - \omega t)$$

(b) Find the total far field expression of electric field $E$ on the $yz$-plane generated by both dipoles.
(c) Let $q_1$ and $q_2$ be real and positive. On the $yz$-plane, if the far field $E$ for $\theta = 60^\circ$ is circularly polarized,

(i) Find the minimum $d$ in terms of $\lambda$.
(ii) What is the ratio of $q_1/q_2$?
(iii) Specify the handness of the circularly polarized wave.