6.642 Continuum Electromechanics
Fall 2008

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A potential sheet, $\Phi(x \ 0) \ \Phi_0 \sin k z \ \text{Re}[\Phi_0 e^{-jk z}]$, is located at $x = 0$ between two dielectric media with dielectric permittivity $\varepsilon_1$ for $-\infty < x < 0$ and $\varepsilon_2$ for $0 < x < d$. A sheet of surface charge, $\sigma_s(x \ d) \ \sigma_0 \cos k z \ \text{Re}[\sigma_0 e^{-jk z}]$, is located at $x = d$ between two dielectric media with dielectric permittivity $\varepsilon_2$ for $0 < x < d$ and $\varepsilon_3$ for $d < x < \infty$. All materials are lossless. The system is of infinite extent in the $y$ direction.

a) Find the complex amplitudes $\sigma_s$ and $\Phi_0$ in terms of $\sigma_0$ and $\Phi_0$.

b) What is the complex amplitude of the potential $\Phi(x \ d)$ along the sheet of surface charge at $x = d$?

c) What is the complex amplitude of the surface charge density along the potential sheet at $x = 0$? (Express your answer in terms of $\sigma_0$, $\Phi_0$, and $\Phi(x \ d)$).

d) What is the space average force per unit area in the $x$ and $z$ directions on the sheet of surface charge at $x = d$? Use the results of part (a) to greatly simplify your answer. Hint: The Maxwell Stress tensor will be the easiest way to solve for the space average free charge and polarization forces in the $x$ and $z$ directions.
Problem 2

An infinitely long cylinder of radius $R$, conductivity $\sigma$, and magnetic permeability $\mu$ is placed in a uniform magnetic field $\vec{H} = z \text{Re}[\hat{H}_0 e^{j\omega t}]$. The region $r > R$ is free space.

The governing equation is the magnetic diffusion equation.

$$\nabla^2 \vec{H} = \mu \sigma \frac{\partial \vec{H}}{\partial t}$$

There is no surface current on the $r = R$ interface.

a) Assume that

$$\vec{H}(r,t) = z \text{Re}[\hat{H}_z(r)e^{j\omega t}]$$

and show that the governing equation is

$$\frac{1}{r} \frac{d}{dr}(r \frac{d\hat{H}_z}{dr}) + j\omega \mu \sigma \hat{H}_z = 0$$

b) Solve part (a) for $\hat{H}_z(r)$ for $r < R$. What boundary conditions must the solution satisfy?

**Hint 1:** The solution to Bessel’s Equation

$$r \frac{d}{dr} \left( r \frac{d\hat{H}_z}{dr} \right) + (k^2 r^2 - n^2) \hat{H}_z = 0$$

is

$$\hat{H}_z(r) = C_n J_n(kr) + \epsilon_n Y_n(kr)$$

where $J_n$ is called a Bessel function of the first kind of order $n$ and $Y_n$ is called the $n^{th}$ order Bessel function of the second kind. Note that $J_n(0)$ is finite while $Y_n(0)$ is infinite. Note also that $k$ has two solutions. Which solution for $k$ can be used to solve for $\hat{H}_z(r)$?

**Hint 2:** $J_0(x) = \frac{x^2}{2} + \frac{x^4}{2^2(2!)^2} - \frac{x^6}{2^2(3!)^2} + \ldots$
c) What is the current density $\bar{J}(r)$?

Hint: $z \frac{d}{dz} J_n(z) = nJ_n(z) = zJ_{n+1}(z)$

d) Plot $H_z(r, t = 0)$ and $\bar{J}(r, t = 0)$ for $\delta R = 0.05, 0.1, 0.25, 0.5, 0.75, 1, \infty$ where $\delta = \sqrt{2/(\omega \mu \sigma)}$ is the skin-depth.