Wave-Based Surveillance

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Define angular spectrum $\overline{E}(\varphi_x, \varphi_y)$ for incoming monochromatic signals

Not to be confused with the radially expanding and diminishing waves characterized by $\overline{E}(\theta, \varphi, R)$

$$\overline{E}(x, y) \approx \int_{4\pi} \overline{E}(\varphi_x, \varphi_y) e^{\frac{j2\pi}{\lambda}(x\varphi_x + y\varphi_y)} d\Omega$$

$$\overline{E}(\varphi_x, \varphi_y) \approx \frac{1}{\lambda^2} \int_A \overline{E}(x, y) e^{-\frac{j2\pi}{\lambda}(x\varphi_x + y\varphi_y)} dx dy \quad \text{For} \quad \varphi_x, \varphi_y \ll \frac{\pi}{2}$$

Equivalently we let $x/\lambda \triangleq x_\lambda$; $y/\lambda \triangleq y_\lambda$
Antenna Aperture Transform Relations and Resolution

Equivalently we let \( x/\lambda \overset{\Delta}{=} x_\lambda \); \( y/\lambda \overset{\Delta}{=} y_\lambda \)

\[
\bar{E}(x_\lambda, y_\lambda) \approx \int_{4\pi} \bar{E}(\varphi_x, \varphi_y) e^{j2\pi(x_\lambda \varphi_x + y_\lambda \varphi_y)} d\Omega
\]

\[
\bar{E}(\varphi_x, \varphi_y) \approx \iint_A \bar{E}(x, y) e^{-j2\pi(x_\lambda \varphi_x + y_\lambda \varphi_y)} dx_\lambda dy_\lambda
\]

Thus: \( \bar{E}(x_\lambda, y_\lambda) \leftrightarrow \bar{E}(\varphi_x, \varphi_y) \)

\[
R_E(\tau_\lambda) \leftrightarrow \left| \bar{E}(\varphi_x, \varphi_y) \right|^2 \propto G(\varphi) \text{ (transmitting)}
\]

where \( R_E(\tau_\lambda) \overset{\Delta}{=} \iint_{-\infty}^{\infty} \bar{E}(r_\lambda) \bar{E}^*(r_\lambda - \tau_\lambda) dx_\lambda dy_\lambda \)

\[
\left( \text{Note: } \bar{E} \text{ is not stochastic} \right)
\]
Single Aperture Resolution Limits

Source image = \( T_A(\bar{\varphi}) = G(\bar{\varphi}) \ast T_B(\bar{\varphi}) \)

\( f_{\varphi}^{\Delta} = \text{cycles per radian (angle)} \)

\( T_A(\bar{f}_{\varphi}) = G(\bar{f}_{\varphi}) \cdot T_B(\bar{f}_{\varphi}) \)

If \( G(\bar{f}_{\varphi}) = 0 \), there is no response in the image spectrum \( T_A(\bar{f}_{\varphi}) \)

Note: \( R_{\tilde{E}}(\bar{\tau}_{\lambda}) \leftrightarrow G(\bar{\varphi}) \leftrightarrow G(\bar{f}_{\varphi}) \) so \( R_{\tilde{E}}(\tau) \cong G(f_{\varphi}) \)
Single Aperture Resolution Limits

Note: \( R_{\vec{E}}(\vec{\tau}_\lambda) \leftrightarrow G(\vec{\varphi}) \leftrightarrow G(\vec{f}_\varphi) \) so \( R_{\vec{E}}(\tau) \cong G(f_{\varphi}) \)

Example:

Note: Zero response to source angular spectral components with spatial frequencies beyond \( f_m = D / \lambda \) cycles/radian
Antenna Responses for Stochastic Signals

Let \( \overline{E}(x_\lambda, y_\lambda, t) \text{(vm}^{-1} \text{ster}^{-1}) = \text{Re}\{\overline{E}(t, x_\lambda, y_\lambda) e^{j\omega t}\} \)

Slowly varying, narrowband random signal

Assume stochastic signals from different directions are uncorrelated (so no systematic intensity variations in aperture).

Then: \( \overline{E}(x_\lambda, y_\lambda, t) \leftrightarrow \overline{E}(\varphi_x, \varphi_y, t) \)

(Double arrow implies irreversibility for two reasons: expectation and magnitude operators used)

\[
E\left[ R_\overline{E}\left(\tau_\lambda\right) \right] \\
\Delta \\
\varphi_\overline{E}(\tau_{x_\lambda}, \tau_{y_\lambda}) \leftrightarrow E\left[ \overline{E}(\varphi_x, \varphi_y, t)^2 \right] \\
\left[ \text{vm}^{-1} \right]^2
\]
Antenna Responses for Stochastic Signals

Then: \( \overline{E}(x_\lambda, y_\lambda, t) \leftrightarrow \overline{E}(\phi_x, \phi_y, t) \) 

(Double arrow implies irreversibility for two reasons: expectation and magnitude operators used)

\[
E \left[ R_E \left( \overline{\tau}_\lambda \right) \right] \triangleq \varphi \overline{E}_{\lambda} \left( \tau_{x_\lambda}, \tau_{y_\lambda} \right) \leftrightarrow E \left[ |\overline{E}(\phi_x, \phi_y, t)|^2 \right]
\]

Can we deduce \( I(\phi_x, \phi_y) \left[ W m^{-2} \text{ster}^{-1} \text{Hz}^{-1} \right] \) from \( \overline{E}(x_\lambda, y_\lambda, t) \)? (Yes)

\[
\frac{\varphi \overline{E}_{\lambda} \left( \tau_{x_\lambda}, \tau_{y_\lambda}, f \right)}{2 \eta_0 B} \leftrightarrow \frac{E \left[ |\overline{E}(\phi_x, \phi_y, f)|^2 \right]}{2 \eta_0 B} = I(\phi_x, \phi_y, f)
\]

\( \frac{(v \text{ m}^{-1})^2 \text{ ohm}^{-1} \text{ Hz}^{-1}}{2(377\Omega)} = W \text{ m}^{-2} \text{ Hz}^{-1} \)
Aperture Field Correlations for a Thermal Source

square source @ $T_B \left( K^\circ \right)$

say $\Omega_s = (0.01 \text{ rad})^2 \approx 10^{-4} \text{ ster}$

Note: $\phi_{E_x}(0, 0) = E\left\{ \left| E(x, y, t) \right|^2 \right\} = \left( \frac{kT_B}{\lambda^2} \Omega_s B \right) S_0(\text{watts}) 2\eta_o$ as expected.
Time and Space Field Correlations (3D)

\[ \phi_{E_x}(\tau_{\lambda_x}, \tau_{\lambda_y}, \tau) / 2\eta_o \left( W m^{-2} \right) \leftrightarrow \phi_{E_x}(\tau_{\lambda_x}, \tau_{\lambda_y}, f) / 2\eta_o \left( W m^{-2} Hz^{-1} \right) \]

\[ \uparrow \left( \tau_{\lambda} \leftrightarrow \phi \right) \]

\[ I_x(\phi, f) = \frac{kT_B(\phi, f)}{\lambda^2} W m^{-2} Hz^{-1} ster^{-1} \]

(one polarization)
Aperture Synthesis

Assume field of size $\tau_{\lambda,\text{max}} \times \tau_{\lambda,\text{max}}$ by $\tau_{\lambda,\text{max}} \times \tau_{\lambda,\text{max}}$ within which two small antennas can be moved independently.

Note $\phi_{\bar{E}}(\bar{\tau}) = \phi_{\bar{E}}^*(\bar{\tau})$

(if stationary w.r.t. $\bar{r}$; i.e., true angular decorrelation)

Therefore, in $\tau_{\lambda}$ space, we need to measure combinations in only two quadrants, e.g., A, B because the conjugates $A^*, B^*$ follow.

We observe $W(\bar{\tau}_\lambda) \cdot \phi_{\bar{E}}(\bar{\tau}_\lambda)$

and retrieve $W(\bar{\varphi}) \ast \left| E(\bar{\varphi}) \right|^2$ where $\left| E(\bar{\varphi}) \right|^2$ is the desired image.
Maximum Antenna Separation Limits Resolution

\[ W(\tau_\lambda) \leftrightarrow W(\varphi) = \]

\[ \tau_{x,\lambda_{\text{max}}} = \frac{L_x}{\lambda} \]

Aperture synthesis: \[ \theta_{3\text{dB}} \approx \frac{\lambda}{2L_x}, \frac{\lambda}{2L_y} \]

Recall, filled aperture: \[ \theta_{3\text{DB}} \approx \frac{\lambda}{L_x}, \frac{\lambda}{L_y} (\eta_A = 1) \]

Origin of difference:
Consider:
Single uniform aperture
Vanishing SNR

Point-source response functions
Good SNR
Circuits for Interferometers

\[ v_1(t) \approx k_1 R_e \left\{ \sqrt{G_1(\phi)} \cdot E(x, y, t) e^{j\omega t} \right\} \]

\[ v_2(t) \approx k_2 R_e \left\{ \sqrt{G_2} \cdot E(x - \tau_x, y - \tau_y, t) e^{j\gamma} e^{j\omega t} \right\} \]

\[ \text{Output} = E\left[v_1(t) v_2(t)\right] = \frac{k_1 k_2}{2} \text{Re}\left\{ \sqrt{G_{12}} \cdot E(x, y, t) \cdot E^*(x - \tau_x, y - \tau_y, t) e^{-j\gamma} \right\} \]

\[ \Delta = G_{12} \]

\[ \text{Recall } E[a(t) b(t)] = \frac{1}{2} R_e \left\{ AB^* \right\}, \quad \text{Re}\{\phi e^{-j\pi/2}\} = \text{Im}\{\phi\} \]
E.G. "T-array"

Recall: \( \frac{\phi \left( \tau_{x\lambda}, \tau_{y\lambda}, f \right)}{2 \eta_o B} \leftrightarrow \frac{|E(\varphi, f)|^2}{2 \eta_o B} = I(\varphi, f) \left( W \text{m}^{-2} \text{s}^{-1} \text{Hz}^{-1} \right) \)

Therefore: \( \left[ \phi_E(\tau_{\lambda}, f) / 2 \eta_o B \right] \cdot W(\tau_{\lambda}) \leftrightarrow I(\varphi, f) * W(\varphi) \)
Image from Discrete Antenna Array

\[
\left[ \phi_{\bar{E}}(\tau, f) / 2\eta B \right] \bullet \mathbf{W}(\tau) \leftrightarrow \mathbf{I}(\varphi, f) \ast \mathbf{W}(\varphi)
\]

\[
\mathbf{W}(\tau_x) = \frac{\lambda}{\Delta \tau_x} \cdot \mathbf{W}(\varphi_x) = \mathbf{I}(\varphi_x) \ast \mathbf{W}(\varphi_x)
\]

Aliased images are confused if they overlap.
Image Aliasing in Synthesized Images

\[
\left( \phi_{\mathcal{E}} \left( \tau_{\lambda}, f \right) / 2\eta_0 B \right) \ast W \left( \tau_{\lambda} \right) \leftrightarrow \Phi \left( \varphi, f \right) \ast W \left( \varphi \right)
\]

\[
I \left( \varphi, f \right) \ast W \left( \varphi \right) \Rightarrow \varphi_y, \varphi_x
\]

\[
\lambda / \Delta \tau_x, \lambda / \Delta \tau_y
\]

\[
\Delta \varphi
\]

To avoid image aliasing, let \( \Delta \tau_x \lesssim \lambda / \Delta \varphi_x \) (source) 

Note that objects in space often are isolated in empty fields, so aliasing is not a problem. Objects imaged on the ground have major aliasing problems, requiring Nyquist-sampled \( \tau_{\lambda} \) plane that puts all aliases in the weak sidelobes of \( G_{AB} \left( \varphi \right) \).

Note: \( \hat{I} \left( \varphi, f \right) = \left[ I \left( \varphi, f \right) \ast W \left( \varphi \right) \right] \ast G_{12} \left( \varphi \right) \)
Aperture Synthesis Using Earth Rotation

Earth rotation moves effective $\tau_x(t)$

Earth spins 1 day per curve

Radio astronomers call $\tau_{\lambda x}, \tau_{\lambda y}$ "u, v"

Choose $\Delta\tau_\lambda$ sufficiently small that no aliasing occurs.

Note: Simple targets that are characterized by the positions or sizes of only a few key features or elements can be deciphered using heavily aliased or burred images.
Recall:

\[ \mathbf{E}(r, t) \leftrightarrow \mathbf{E}(\psi, t) \]

\[ \mathbf{E}\left[ R_\mathbf{E}\left( \tau_\lambda \right) \right] = \phi_\mathbf{E}\left( \tau_\lambda \right) \leftrightarrow \mathbf{E}\left\{ \left| \mathbf{E}(\Psi, t) \right|^2 \right\} \propto I(\Psi), T_A(\Psi) \]
Simple Adding Interferometers

$$\bar{v}_o(\psi) = \bar{a}^2 + \bar{b}^2 + 2\bar{a}\bar{b}$$

(overbars mean "time average" here)

Point source response

$$\bar{v}_o(\psi_x)$$

$\theta_B$ (single antenna)

First null

$$\lambda/L \cong T$$
Simple Adding Interferometers

\[ \psi_{null} = \sin^{-1}\left(\frac{\lambda}{2L}\right) \]

\[ \cong \frac{\lambda}{2L} \text{ if } \lambda \ll L \]

Complex fringe visibility \( v \) is

\[ v \triangleq \frac{v_{max} - v_{min}}{v_{max} + v_{min}} \exp\left\{ j\frac{2\pi T_1}{T} \right\} \propto E\left(\tau_{\lambda}\right) \leftrightarrow T_A\left(\psi\right) \]

Spread source response

DC: \( a^2 + b^2 \)

AC: \( 2ab \)
Interferometry as Fourier Analysis

Recall: \[ \phi_A (\tau_\lambda) = R_E (\tau_\lambda) \ast \phi_E (\tau_\lambda) \]
\[ T_A (\psi) = G (\psi) \ast T_B (\psi) \]
\[ T_A (f_\psi) = G (f_\psi) \ast T_B (f_\psi) \]

observed = antenna times signal

Example: 2 small duplicate antennas separated by \( D_\lambda \) in the \( x \) direction

2-element adding interferometer

\[ G_{\cos} (\psi_x) \propto 1 + \cos D_\lambda \psi_x \]
\[ G_{\sin} (\psi_x) \propto 1 + \sin D_\lambda \psi_x \]
Interferometry as Fourier Analysis

2-element adding interferometer

\[
\begin{align*}
G_{\cos}(\psi_x) & \propto 1 + \cos D_{\lambda} \psi_x \\
G_{\sin}(\psi_x) & \propto 1 + \sin D_{\lambda} \psi_x
\end{align*}
\]

Interferometer directly measures Fourier components of source

Effect of finite bandwidth:

Fringe patterns for all frequencies (colors) add in phase to create "white fringe"

\[
\begin{align*}
f_o + \Delta (\text{Hz}) & \quad + \quad f_o (\text{Hz}) & \quad + \quad f_o - \Delta (\text{Hz}) \\
\psi_x & \quad + \quad \psi_x & \quad + \quad \psi_x
\end{align*}
\]

\[\Delta \psi_x \propto \frac{1}{B}\]

B = 2\Delta

A delay line in one interferometer arm can redirect the strong white fringe in other directions.
Broad-Bandwidth Effects in Interferometers

\[ E(x, y, t) \text{ (the "slowly varying" part)} \]

may vary rapidly enough that the offset time \( L \sin \frac{\varphi_x}{c} \) is significant

\[
E[v_1(t)v_2(t)] = \frac{k_1k_2}{2} G_{12}(\overline{\phi}) E\left[ \Re \left\{ E\left( x, y, t + \frac{L \sin \varphi_x}{2c} \right) \right\} \right]
\]

\[
E^* \left( x - \tau_x, y - \tau_y, t - \frac{L \sin \varphi_x}{2c} \right) \cdot e^{j\omega L \sin \varphi_x / c} e^{-j\gamma} \left[ e^{j\omega(t+L \sin \varphi_x / 2c)} e^{-j\omega(t-L \sin \varphi_x / 2c)} = e^{j\omega L \sin \varphi_x / c} \right]
\]
In broadband optical interferometer all colors contribute to central "white" fringe; sidelobe fringes appear colored.

Therefore \( \varphi_{x_{null}} = \sin^{-1}\left(\frac{c}{2LB}\right) \approx \frac{c}{2LB} \) for \( \varphi_x \approx 1 \)

e.g. \( \varphi_{x_{null}} \approx \frac{3 \times 10^8}{2 \times 10 \times 10^7} = 1.5 \) radians for \( L = 10 \) m, \( B = 10 \) MHz

If \( B = 1 \) GHz, and \( L = 100 \) m, then \( \varphi_{x_{null}} = 1.5 \) mrad \( \approx 5 \) arc min

\( B = 3 \times 10^{14} \) Hz and \( L = 100 \) m, then \( \varphi_{x_{null}} = 10^{-3} \) arc sec.
Dicke Adding Interferometer

\[ y(\psi) \text{ point source response} \]

+1 Dicke

-1

\[ \langle a^2 + b^2 \pm 2ab \rangle \]

(Also called "lobe-switching" interferometer)

This circuit cancels D.C. term, leaving only \( \langle 4ab \rangle \)
as source traverses beam ⇒ \( \psi \)

"Adding" interferometer

Dicke switch oscillator

\[ y(\varphi) = (\pm a + b)^2 \]

Can add second adder and square-law device operating on \( a \) and \(-jb\) to yield sine terms in Fourier expansion

\[ \int_{\tau} \left( a^2 + b^2 + 2ab \right) - \left( a^2 - 2ab + b^2 \right) = 4ab \]
Dicke Adding Interferometer

Yields $\phi(\tau_\lambda)$ for even a stationary source

Note: If no Dicke switch, have gain fluctuation vulnerability

"Multiplying" interferometer

$\text{Re}\left\{\phi(\tau_\lambda)\right\}$

$\text{Im}\left\{\phi(\tau_\lambda)\right\}$
“Lobe-Scanning” Interferometer

Lobes are scanned at $\omega_m$, demodulated, and averaged to yield $\phi(\tau_{\lambda})$

Because all bias and large-scale sources yield no fine-scale response, can integrate long times seeking fine structure, e.g., $10^{-3}$ Jansky point sources like stars (can measure stellar diameters at $\sim 10^{-3}$ arc sec)

\[ \propto G_{12} \Re \left\{ \phi(\tau_{\lambda}) \right\} \quad \propto G_{12} \Im \left\{ \phi(\tau_{\lambda}) \right\} \]
Cross-Correlation Interferometer Spectrometer

Let $a(t) \triangleq a_1$

$a(t - \tau) \triangleq a_2$

$c_+ = a_1a_2 + b_1b_2 + a_1b_2 + a_2b_1$

$c_- = -a_1a_2 - b_1b_2 + a_1b_2 + a_2b_1$

Note: if $a = b$, $\phi(-\tau_\lambda, f) \rightarrow \phi(0, f) = S(f)$

if $a \perp b$, $\phi(-\tau_\lambda, f) \rightarrow 0$
Alternate Cross-Correlation Interferometer Spectrometer

Note: Figures omit down-converters and bandlimiting filters
Mechanical Long Distance Phase Synchronization

For $L \gg \lambda$, L.O. synchronization can be degraded by random phase variations in path length $L$ between two (or more) sites (due principally to thermal and acoustic variations)

One standard solution:

"Line stretcher" varies path so that $\Delta \phi \approx 0$

Communications and telemetry systems can similarly be phase synchronized
Synchronizing with Remote Atomic Clocks

If distance L is too great to synchronize L.O.'s, then we can use a remote clock, e.g. "very-long baseline" interferometry, "VLBI"

If clocks perfect: cross-correlate to find time offset, correct for it, then correlate the signals, albeit with an unknown fixed phase offset $\phi_0$ [unless reference source (in the sky) or phase is available].

If clocks imperfect and delays each way are identical: at site A measure delay between clock B and A; do the same at B, and subtract results to yield twice the clock offsets. Use this offset to align A and B data streams.
Synchronizing with Remote Atomic Clocks

Alternative approaches if clocks and transmissions are imperfect:

a) Track and correct phase shifts:

Then average $\hat{\phi}(\tau_\lambda, f)$

Phase $\beta$

Example: A $10^{-12}$ cesium clock drifts $2\pi$ in $\sim 100$ sec, so $\hat{\phi}(\tau_\lambda, f)$ might be computed for 5–10 sec blocks before averaging; then only $|\hat{\phi}|$ is known.

b) Same, but set phase using strong resonant line point source,

c) or separable point source in space, modulation, etc.
Multiband Synchronization of Clocks

Use multiple frequencies for wideband sources:

Which fringe F is source on?

Switch across all f's within coherence time of clock.

Observed bands (collectively adequate)
**Phaseless Interferometry**

Hanbury-Brown and Twiss

Visible interferometer at Narrabri, Australia

\[
\begin{bmatrix}
\text{T}_A \\
\text{B} \\
2 \\
\text{LPF} \\
\int \tau \\
\phi_E \left( \overline{\tau}_\lambda \right)^2 \\
0 \\
\end{bmatrix}
\]

\[
\text{For } T_R >> T_A, \quad \frac{v_{\text{rms}}}{\langle v_0 \rangle} \approx \frac{T_R^2}{T_A \sqrt{2W \tau}}
\]

One might (wrongly) think photodetectors would lose all phase information and ability to measure source structure at \( \lambda/D \) resolution.

Recall: \( E[aabb] = \alpha^2 b^2 + 2\alpha b^2 \), where \( \alpha b \) is \( \phi_E \left( \overline{\tau}_y \right) \) here.
Phaseless Recovery of Source Structure

Recall: $E[aabb] = a^2 b^2 + 2ab^2$, where $ab$ is $\phi_E(\tau_y)$ here.

Recall: $E(x, y) \iff E(\psi)$

Purely real if source is even function of position, allowing perfect source reconstruction.

$|\phi_E(\tau_{\lambda})|^2 \iff R \frac{\mid E(\psi)\mid^2}{E(\psi)} \left( \Delta \psi \right)$
Phaseless Interferometer Interpretation: Independent Radiators

Source, independent thermal radiators g and h

a, b are uncorrelated if $\Delta \phi_a - \Delta \phi_b \gtrsim 2\pi$

[$\Delta \phi_a$ is $\Delta \phi$ at "a" for rays g, h]; or if

$$\frac{\phi_s}{2} = \phi \gtrsim \left(\frac{\lambda}{2}\right)/L.$$

Thus a, b decorrelated if $\phi_s \gtrsim \lambda/L$. 
Phaseless Interferometer Diffraction-Limited Source

If $\theta > \theta_B \approx \lambda/D$, then $a$ and $b$ are ~uncorrelated.

Therefore decorrelated if $D\theta \lesssim \lambda$

or if $DL/R \lesssim \lambda$ since $\theta \approx L/R$

or if $\phi_s \lesssim \lambda/L$ since $\phi_s \approx D/R$
Radar Equation

Issues: Signal design
Processor design
Antenna

Propagation, absorption, refraction, scintillation, scattering, multipath

Scattering

Issues:

\[
P_{\text{rec}} = \frac{P_t}{4\pi R^2} \cdot G_t \cdot \frac{\sigma}{4\pi R^2} \cdot A_t = P_t \left( \frac{G\lambda}{4\pi R^2} \right)^2 \frac{\sigma}{4\pi} \text{ Watts}
\]

\[
\text{Wm}^{-2} \text{ at transmitter}
\]

\[
\text{Wm}^{-2} \text{ at target}
\]

\(\sigma\) "scattering cross-section" is equivalent capture cross-section for a target scattering isotropically.
Radar Scattering Cross-Section

\[ \sigma \text{ "scattering cross-section" is equivalent capture cross-section for a target scattering isotropically} \]

Note: Corner reflector can have \( \sigma \gg \) size of target

Biastatic radars:

If target is unresolved, \( P_{\text{rec}} \propto 1/R_1^2R_2^2 \)
If target is resolved by the transmitter, \( P_{\text{rec}} \propto 1/R_2^2 \)

Note resolution enhancement:

\[ P_{\text{rec}} \propto R^{-4}G^2 \text{ where } G^2(\theta) \text{ has a narrower beam than } G(\theta) \]
Target Scattering Laws

1) Specular

2) Scintillating

3) Faceted

4) Lambertian

\[ \delta \ll \lambda \Rightarrow \text{nulls} \]

\[ P(\theta) \]

\[ \cos \theta \propto \text{power scattered} \]

(geometric projection only)
Target Scattering Laws

5) Random Bragg Scattering (frequency selective)
At Bragg angles $\Delta \text{Path}_{1,2} = n \cdot 2\pi$  
$n = 0, \pm 1, \pm 2, \ldots$

Path 2 between phase fronts A and B

6) Sub-Surface Inhomogeneities

~ random Bragg scattering

Quasi-periodic surface

e.g. rocks
Target Range-Doppler Response

Narrowband Pulsed Radar

\[ \sigma(t, f_0) \]

Impulse response function for a deep target

e.g.

Moon

Deep Target

CW Radar

"Spins"

Return Doppler, shifted to \( f_0 + \Delta f \)
Range-Doppler Response for a CW Pulse

Note north-south ambiguity