Wave Propagation

Molecular line absorption by gases:

- permanent electric dipole (H₂O, CO)
- permanent magnetic dipole (O₂)
- unpolarized (N₂) (collision-induced dipoles)

Quantized energy levels: $E_i - E_j = hf$

- electronic states
- vibrational states
- rotational states
- nuclear spin states

Visible, UV, x-ray

IR - µW

Visible I.R.

r.f. → audio
Molecular Lines in Gases

Quantized energy levels: \( E_i - E_j = hf \)

Probability of radiation = \( A + B \rho_f \)

\begin{align*}
\text{“Einstein ‘A’ coeff.”} & \uparrow \\
\text{“Einstein ‘B’ coeff.”} & \uparrow \\
\text{radiation intensity (energy density)} & \uparrow \\
\end{align*}

Probability of absorption = \( B\rho_f \)

\[
A/B = 8\pi hf^3/c^3
\]

Collisions and radiation compete to control level populations. In equilibrium, kinetic and radiation temperatures are equal.
Molecular Line Shape

Broadening: intrinsic, collisional, Doppler

Einstein “A” yields spontaneous emission, limiting state lifetime $T$; intrinsic linewidth $\cong 1/T$

\[ \Delta \omega = \text{linewidth} \propto \text{number of collisions/sec} \]

“Pressure broadening” or “collision broadening” $\cong 2$ GHz at standard temperature and pressure (STP) for $\text{O}_2$, $\text{H}_2\text{O} < 1$ THz

Doppler broadening has thermal $\left( \frac{1}{2} m v^2 \cong \frac{3}{2} kT \right)$, turbulent (random), and systematic components
Overlapping Spectral Lines

Superposition characterizes the cumulative absorption by independent spectral lines, except for certain single molecules.

For a single molecule with collision-coupled states, total absorption is generally greater than sum of line absorptions between coupled lines, and less outside. Such coupled lines coherently “interfere” (e.g. 60-GHz oxygen band).
Refractive Effects

The permittivity $\varepsilon(f)$ of a medium is related in part to the absorption coefficient $\alpha(f)$ by the Hilbert transform; $\alpha(f)$ is related to the imaginary part of $\varepsilon(f)$.

Atmospheric water vapor scale height = ~2 km
Atmospheric density scale height $\approx$ ~8 km
So humidity-based refractive effects are mostly a lower tropospheric phenomenon ($\lesssim$ 8 km).

Thermal inhomogeneities are turbulent in the boundary layer (first few hundred meters or more) and near convective instabilities, and are more layered at higher altitudes.

Humidity variations often dominate radio refraction, while density variations dominate optical propagation.
Optical telescopes have ~1 arc-sec “seeing” on good nights (2” – 10” in Boston is typical); the best mountain days may yield ~0.4 arc sec., where “seeing” is the blur spot size, not absolute refraction.
Refractive Effects

The radio index of refraction $n$ is given by:

$$ (n - 1)10^6 = \frac{79}{T}(p + 4800 \ \text{e/T}) $$

where $T$ is °K, and $p$ and $e$ are total and partial water vapor pressures (mb).

Ducting can occur in cold or humid layers of air, or in under-ionized ionospheric layers. Acoustic ducting can occur in cool or salty ocean layers.

Refractive seeing beyond the horizon can be $\gtrsim 30$ arc minutes on RF, and less at optical frequencies.

Fading caused by interfering multipath: paths of different length cause different frequencies to cancel out or “fade.”
Plasmas can have both neutral and ionized components. The ionosphere has \( n_e \approx 10^7 - 10^{12} \text{ (m}^{-3}\text{)} \) from \( \sim 50 \) to 5000 km altitude. Electron density \( n_e \text{(max)} \) is \( \sim 100 - 400 \) km.

Plasma frequency:

\[
\omega_p = \sqrt{\frac{n_e q^2}{m \varepsilon_0}} \text{ (rs}^{-1}\text{)} \quad \text{where} \quad m = \frac{m_e m_i}{m_e + m_i} \approx m_e
\]

\[
\varepsilon = \varepsilon_0 \left(1 - \frac{\omega_p^2}{\omega^2}\right) \quad \text{q = electron charge}
\]

Evanescent waves only, if \( \omega < \omega_p \)

Propagation delay:

**phase velocity** \( v_p = c\sqrt{1 - \left(\frac{\omega_p}{\omega}\right)^2} > c \) \( \text{for } f > f_p \)

**group velocity** \( v_g = c\sqrt{1 - \left(\frac{\omega_p}{\omega}\right)^2} < c \) \( \left(v_g v_p = c^2 \text{ here}\right) \)
Refraction and Absorption by Plasmas

Refraction by plasmas:
- \( \omega > \omega_p \) refraction is governed by Snell’s Law
  \[ \frac{\sin \theta_i}{\sin \theta_t} = \frac{v_{p_t}}{v_{p_i}} \]
- \( \omega < \omega_p \) evanescent waves, total reflection

Absorption by plasmas:
- Transient electric dipole emits and absorbs
- \# collisions \( \propto n_e^2 \) (weak in ionosphere)
Magnetized Plasmas

The EM interaction and Faraday rotation become strong near the electron and ion cyclotron resonances, $\omega_c = qB_o/m$.
Scattering and Absorption by Dielectric Spheres

EM wave $\vec{E}$, sphere (e.g. raindrop)

$D \ll \frac{\lambda_0}{2\pi} \sqrt{\frac{\varepsilon_0}{\varepsilon}}$ ⇒ "Rayleigh scattering"

$D \approx \frac{\lambda_0}{2\pi}$ ⇒ "Mie scattering"

$D \gg \frac{\lambda_0}{2\pi}$ ⇒ Geometric scattering

Mie is multimode:

null

dipole re-radiation pattern

induced electric dipole

log $(\sigma_s)$
slope = 4

Rayleigh

Geometric

Mie

slope = 4

$\pi a^2$

$0.25\pi a^2$

$4\pi a^2$
Scattering and Absorption by Dielectric Spheres

Rayleigh regime \((\lambda >> 2\pi D \varepsilon / \varepsilon_0)\): (constant \(\varepsilon\))

\[
\sigma_s \propto (a/\lambda)^6 \lambda^2 \quad \text{scattering cross-section, } \alpha_{\text{scattering}} \left(\text{dBm}^{-1}\right) \propto f^4
\]

\[
\sigma_a \propto (a/\lambda)^3 \lambda \quad \text{absorption cross-section, } \alpha_{\text{absorption}} \left(\text{dBm}^{-1}\right) \propto f^2
\]

Cloud absorption \(\lesssim 85\) GHz (Rayleigh region):

\[\gamma_{\text{CLD}} \left(\text{nepers cm}^{-1}\right) \approx m \cdot 10^{[0.0122(291-T)-6]} \lambda^2\]

where \(m = \text{g/m}^3\) liquid water, \(\lambda = \text{wavelength cm, } T = K\)

Albedo \(\lesssim 0.8, f_{\text{max scat.}} \approx 100 - 150\) GHz

\(\therefore T_B \gtrsim 70\)K as seen from space

\((\text{Albedo } \triangleq \text{ reflectivity, all angles})\)

Rain attenuation \(> 30\)dB sometimes