Lecture 16 - p-n Junction (cont.)

March 12, 2007

Contents:

1. Non-ideal and second-order effects

Reading assignment:

del Alamo, Ch. 6, §6.4

Announcements:

Quiz 1: **March 13**, 7:30-9:30 PM; lectures #1-12 (up to SCR-type transport). Open book. **Calculator required.**
Key questions

• What happens if there is SCR generation and recombination in a pn diode?

• If the doping distribution in a p-n junction is non-uniform, is the basic operation of the diode changed in a fundamental way?

• What happens to the C-V characteristics, I-V characteristics, and the dynamics of a p-n diode with non-uniform doping distributions?
1. Non-ideal and second-order effects

- **Space-charge generation and recombination**

In real devices, non-ideal $I - V$ characteristics often seen:

Anomalies often due to:

- recombination through traps in SCR (in forward bias)
- generation through traps in SCR (in reverse bias)
Simple model for SCR generation and recombination.

Starting point: trap-assisted G/R rate equation:

\[
U_{tr} = \frac{np - n_0p_0}{\tau_{ho}(n + n_i) + \tau_{co}(p + n_i)}
\]

In SCR:

\[
np = n_i^2 \exp \frac{qV}{kT}
\]

Then:

\[
U_{tr,SCR} = \frac{n_i^2(\exp \frac{qV}{kT} - 1)}{\tau_{ho}n + \tau_{co}p + (\tau_{ho} + \tau_{co})n_i}
\]

SCR G/R current:

\[
J_{SCR} = q \int_{-x_p}^{x_n} U_{tr} dx
\]

Since \( n \) and \( p \) changing quickly with \( x \) in SCR, no analytical solution.
Analytical model:

\[ U_{tr}\big|_{SCR} = \frac{n_i^2 (\exp \frac{qV}{kT} - 1)}{\tau_{ho} n + \tau_{co} p + (\tau_{ho} + \tau_{co}) n_i} \]

Since \( np \) constant, point of SCR with highest \( U_{tr} \) where:

\[ \tau_{ho} n = \tau_{co} p \]

At that point:

\[ U_{tr}\big|_{SCR, max} = \frac{n_i}{2 \sqrt{\tau_{co} \tau_{ho}}} (\exp \frac{qV}{2kT} - 1) \]

Use this across entire SCR → upper limit to current:

\[ J_{SCR, max} = \frac{qn_i x_{SCR}}{2 \sqrt{\tau_{co} \tau_{ho}}} (\exp \frac{qV}{2kT} - 1) \]
\[ J_{SCR,max} = \frac{q n_i x_{SCR}}{2 \sqrt{\tau_{eo} \tau_{ho}}} \left( \exp \frac{qV}{2kT} - 1 \right) \]

★ Key dependencies:

- **Forward bias:** SCR recombination \( \sim \exp \frac{qV}{2kT} \)
in contrast with QNR recombination \( \sim \exp \frac{qV}{kT} \)
- **In practice,** \( 1 < n < 2 \) for SCR recombination
- **SCR G/R \( \sim n_i \), in contrast with QNR G/R \( \sim n_i^2 \)
  \( \Rightarrow E_a(SCR) \simeq E_g/2 \), in contrast with \( E_a(QNR) \simeq E_g \)
- **SCR G/R highly process sensitive:** small SCR G/R current hallmark of “clean” process,
- **Reverse bias:** SCR generation \( \sim x_{SCR} \Rightarrow I \sim \sqrt{|V|} \)
Two diodes in weblab:
□ Non-uniform doping level

”Real” p-n diodes have doping profiles that are highly non-uniform:

- No major differences in operation of pn diode, *e.g.* rectifying characteristics of p-n diode.
- Some qualitative differences, *e.g.*, the voltage dependence of the C-V characteristics.
- Need to revise computation of $I$, $C$, and minority carrier time constants.
* Depletion capacitance

p-n junction electrostatics affected by doping non-uniformity.

Treat simple case: *linearly graded junction*:

\[ N(x) = N_D - N_A = ax \]

Do depletion approximation with:

\[ x_n = x_p = \frac{x_{SCR}}{2} \]
• Volume charge density:

\[
\rho(x) = qa x \quad \text{for} \quad -\frac{x_{SCR}}{2} \leq x \leq \frac{x_{SCR}}{2} \\
\rho(x) \approx 0 \quad \text{outside}
\]

• Electric field:

\[
\mathcal{E}(x) = \frac{qa}{2\varepsilon} [x^2 - (\frac{x_{SCR}}{2})^2] \quad \text{for} \quad -\frac{x_{SCR}}{2} \leq x \leq \frac{x_{SCR}}{2} \\
\mathcal{E}(x) \approx 0 \quad \text{outside}
\]

• Electrostatic potential distribution \((\phi(x) = 0) = 0\):

\[
\phi(x) = \frac{qa}{6\varepsilon} [3(\frac{x_{SCR}}{2})^2 x - x^3] \quad \text{for} \quad -\frac{x_{SCR}}{2} \leq x \leq \frac{x_{SCR}}{2}
\]
$x_{SCR}$ determined by demanding that:

$$\phi\left(\frac{x_{SCR}}{2}\right) - \phi\left(-\frac{x_{SCR}}{2}\right) = \phi_{bi} - V$$

Then:

$$x_{SCR} = \left[\frac{12\epsilon(\phi_{bi} - V)}{qa}\right]^{1/3}$$

With:

$$\phi_{bi} = \frac{kT}{q} \ln \frac{n_o(x_{SCR}/2)}{n_o(-x_{SCR}/2)} = 2 \frac{kT}{q} \ln \frac{a x_{SCR}}{2n_i}$$

These two equations need to be solved iteratively.

Capacitance:

$$C = \left[\frac{qa\epsilon^2}{12(\phi_{bi} - V)}\right]^{1/3} = \frac{C(V = 0)}{(1 - \frac{V}{\phi_{bi}})^{1/3}}$$

With:

$$\frac{1}{C^3} = \frac{12(\phi_{bi} - V)}{qa\epsilon^2}$$
Experiments:

In general, depletion capacitance well modeled by:

$$C = \frac{C_o}{(1 - \frac{V}{\phi_{bi}})^m}$$

$C_o$ is value of $C$ at $V = 0$.

- $m = 0.5$ for ideal abrupt junction
- $m = 0.33$ for ideal gradual junction

Figure 2 on page 143 in
**Current**

- Non-uniform doping distribution does not affect basic physics of minority carriers.
- Electric field associated with non-uniform doping distribution affects overall carrier velocity: drift added to diffusion.
- Computation of current density and dominant minority carrier time constant more complex.

Electric field affects current and transit time by aiding or opposing minority carrier diffusion.

- "Downgoing" doping profile aids diffusion $\rightarrow I \uparrow \tau_t \downarrow$
- "Upgoing" doping profile opposes diffusion $\rightarrow I \downarrow \tau_t \uparrow$

Cite as: Jesús del Alamo, course materials for 6.720J Integrated Microelectronic Devices, Spring 2007. MIT OpenCourseWare (http://ocw.mit.edu/), Massachusetts Institute of Technology. Downloaded on [DD Month YYYY].
Analytical solution obtainable only in case of \textit{transparent} or short region: minority carrier recombination takes place at surface.

For p-region:

\begin{equation}
J_e = qn'\mu_e\mathcal{E}_o - qD_e\frac{dn'}{dx}
\end{equation}

Since:

\begin{equation}
\mathcal{E}_o = \frac{kT}{qN_A} \frac{1}{dx}
\end{equation}

Then:

\begin{equation}
J_e = q\frac{D_e}{N_A} \frac{d(n'N_A)}{dx}
\end{equation}

Integrate across p-QNR:

\begin{equation}
\int_{-w_p}^{-x_p} J_e \frac{N_A}{qD_e} dx = \int_{-w_p}^{-x_p} \frac{d(n'N_A)}{dx} dx = n'N_A|_{-x_p} - n'N_A|_{-w_p}
\end{equation}
B.C.'s:

- at junction \((x = -x_p)\):

\[
n'(-x_p) = \frac{n_i^2}{N_A(-x_p)}(\exp \frac{qV}{kT} - 1)
\]

- at surface \((x = -w_p)\) with \(S = \infty\):

\[
n'(-w_p) = 0
\]

If recombination mainly takes place at surface, \(J_e\) independent of \(x\), and:

\[
J_e(-x_p) = \frac{q n_i^2}{\int_{-w_p}^{-x_p} \frac{N_A}{D_e} dx} \left( \exp \frac{qV}{kT} - 1 \right) \simeq \frac{q n_i^2 < D_e >}{\int_{-w_p}^{-x_p} N_A dx} \left( \exp \frac{qV}{kT} - 1 \right)
\]

Since \(D_e\) is slow function of \(N_A\), oftentimes

\[
\int_{-w_p}^{-x_p} N_A dx
\]

referred to as \textit{Gummel number} → to first order, only integrated doping concentration counts to set the current!

Similar equation for n-type side.
*Dynamics:* calculation of transit time in "transparent" region.

Go back to:

\[ J_e = q \frac{D_e}{N_A} \frac{d(n'N_A)}{dx} \]

Integrate up to \( x \):

\[ \int_{-w_p}^{-x} J_e \frac{N_A}{qD_e} dx = \int_{-w_p}^{-x} \frac{d(n'N_A)}{dx} dx = n'N_A|_{-x} - n'N_A|_{-w_p} \]

If \( S = \infty, n'(-w_p) = 0 \), and:

\[ n'(x) = \frac{J_e(-x_p)}{qN_A(x)} \int_{-w_p}^{-x} \frac{N_A}{D_e} dx \]

Total minority carrier charge:

\[ Q_p = q \int_{-w_p}^{-x_p} n'(x) dx = J_e(-x_p) \int_{-w_p}^{-x_p} \frac{1}{N_A} \left( \int_{-w_p}^{-x} \frac{N_A}{D_e} dx \right) dx \]

Diffusion capacitance:

\[ C_{dp} = \frac{dQ_p}{dV} = \frac{q}{kT} J_e(-x_p) \int_{-w_p}^{-x_p} \frac{1}{N_A} \left( \int_{-w_p}^{-x} \frac{N_A}{D_e} dx \right) dx \]

Then:

\[ \tau_{tp} = \int_{-w_p}^{-x_p} \frac{1}{N_A} \left( \int_{-w_p}^{-x} \frac{N_A}{D_e} dx \right) dx \]
Key conclusions

• SCR generation and recombination dominates in low forward bias and in reverse bias.

• Key characteristic of SCR recombination in forward bias:

\[ J_{SCR} \propto \exp \left( \frac{qV}{nkT} \right) \quad \text{with } 1 < n < 2 \]

• Key characteristic of SCR generation in reverse bias:

\[ J_{SCR} \propto \sqrt{|V|} \]

• Non-uniformly doped regions do not affect basic operation of pn junction.

• Exponent of dependence of depletion capacitance with voltage is function of doping distribution:

\[ m = 0.5 \text{ for abrupt junction} \]
\[ m = 0.33 \text{ for linearly graded junction.} \]

• Integrated doping concentration sets minority carrier current in ”transparent” or ”short” non-uniformly doped QNR.

• Minority carrier transit time through non-uniformly doped QNR depends on details of impurity profile.