Lecture 27 - The "Long"
Metal-Oxide-Semiconductor Field-Effect
Transistor (cont.)

April 13, 2007

Contents:

1. Charge-voltage characteristics of ideal MOSFET (cont.)
2. Small-signal behavior of ideal MOSFET
3. Short-circuit current-gain cut-off frequency, $f_T$

Reading assignment:

del Alamo, Ch. 9, §§9.5 (9.5.2), 9.6
Key questions

• What are the capacitances associated with the inversion layer charge?

• What is the topology of a small-signal equivalent circuit model for the MOSFET?

• What are the key bias and geometry dependencies of all small-signal elements in the model?

• How does one characterize the frequency response of a transistor?
1. Charge-voltage characteristics of ideal MOSFET

(continuation)

- Inversion charge:

\[ Q_I = W \int_0^L Q_i(y) \, dy \]

For \( V_{GS} > V_T \):

\[ Q_I = W \int_0^{V_{DS}} Q_i(V) \frac{dy}{dV} \, dV \]
\[ Q_I = W \int_0^{V_{DS}} Q_i(V) \frac{dy}{dV} dV \]

From channel current equation, we have:

\[ \frac{dy}{dV} |_{V} = -\frac{W \mu_e}{I_D} Q_i(V) \]

Then:

\[ Q_I = -\frac{W^2 \mu_e}{I_D} \int_0^{V_{DS}} Q_i^2(V) dV \]

Now use charge-control relationship:

\[ Q_i(V) = -C_{ox}(V_{GS} - V - V_T) \]

Finally get:

\[ Q_I = -\frac{2}{3} W LC_{ox} \frac{(V_{GS} - V_T)^2 + (V_{GS} - V_T)(V_{GD} - V_T) + (V_{GD} - V_T)^2}{(V_{GS} - V_T) + (V_{GD} - V_T)} \]
\[ Q_I = -\frac{2}{3} W L C_{ox} \frac{(V_{GS} - V_T)^2 + (V_{GS} - V_T)(V_{GD} - V_T) + (V_{GD} - V_T)^2}{(V_{GS} - V_T) + (V_{GD} - V_T)} \]

Evolution of \( Q_I \) with \( V_{DS} \):

Used fundamental charge control relationship \( \Rightarrow \) expression only valid in linear regime.

For small \( V_{DS} \):

\[ Q_I \approx -W L C_{ox}(V_{GS} - V_T) \]
For saturation: set \( V_{DS} = V_{DS_{\text{sat}}} = V_{GS} - V_T \) in linear regime expression and get:

\[
Q_I = -\frac{2}{3} W L C_{ox} (V_{GS} - V_T)
\]

Reduction of \(|Q_I|\) towards saturation is another manifestation of channel debiasing:
\[ C_{gsi} = -\frac{\partial Q_I}{\partial V_{GS}}|_{V_{GD}} = \frac{1}{2}WL C_{ox}(V_{GS} - V_T) \frac{V_{GS} - V_T - \frac{2}{3}V_{DS}}{(V_{GS} - V_T - \frac{1}{2}V_{DS})^2} \]

\[ C_{gdi} = -\frac{\partial Q_I}{\partial V_{GD}}|_{V_{GS}} = \frac{1}{2}WL C_{ox}(V_{GS} - V_{DS} - V_T) \frac{V_{GS} - V_T - \frac{1}{3}V_{DS}}{(V_{GS} - V_T - \frac{1}{2}V_{DS})^2} \]

Expressions valid in linear regime.

Note that for small \( V_{DS} \):

\[ C_{gsi} \simeq C_{gdi} \simeq \frac{1}{2}WL C_{ox} \]

For saturation regime, set \( V_{DS} = V_{DSat} = V_{GS} - V_T \) and get:

\[ C_{gsi} = \frac{2}{3}WL C_{ox} \]

\[ C_{gdi} = 0 \]
Evolution of $C_{gsi}$ and $C_{gdi}$ with $V_{DS}$:

- Linear regime (small $V_{DS}$): uniform inversion layer charge:
  \[ C_{gsi} \approx C_{gdi} \approx \frac{1}{2} WLC_{ox} \]

- Saturation regime ($V_{DS} > V_{DSsat}$): channel pinched-off:
  \[ C_{gs} \approx \frac{2}{3} WLC_{ox} \quad C_{gd} \approx 0 \]
2. Small-signal behavior of ideal MOSFET

In many applications, interested in response of device to *small signal* applied on top of bias:

\[
i_D = I_D + i_d = \frac{v_{gs} - V_{BS}}{g_{ms}} + \frac{v_{ds} - V_D}{r_d} = \frac{v_{gs} - V_{BS}}{g_{ms}} + \frac{v_{ds} - V_D}{r_d}
\]

Key points:

- Small signal is *small* ⇒ non-linear device behavior becomes linear.
- Can separate response of MOSFET to bias and small signal.
- Since response is linear, *superposition* applies ⇒ effects of different small-signals independent from each other.

Mathematically:

\[
i_D(V_{GS}, V_{DS}, V_{BS}; v_{gs}, v_{ds}, v_{bs}) \simeq I_D(V_{GS}, V_{DS}, V_{BS}) + i_d(v_{gs}, v_{ds}, v_{bs})
\]

and

\[
i_d(v_{gs}, v_{ds}, v_{bs}) = i_d(v_{gs}) + i_d(v_{ds}) + i_d(v_{bs})
\]
\( i_d \) linear on small-signal drives:

\[
i_d \approx g_m v_{gs} + g_d v_{ds} + g_{mb} v_{bs}
\]

Define:

- \( g_m \equiv \text{transconductance} \ [S] \)
- \( g_d \equiv \text{output or drain conductance} \ [S] \)
- \( g_{mb} \equiv \text{back transconductance} \ [S] \)

Equivalent circuit model representation:

Approach to computing \( g_m, g_d, \) and \( g_{mb} \):

- \( g_m \approx \left. \frac{\partial I_D}{\partial V_{GS}} \right|_{V_{DS},V_{BS}} \)
- \( g_d \approx \left. \frac{\partial I_D}{\partial V_{DS}} \right|_{V_{GS},V_{BS}} \)
- \( g_{mb} \approx \left. \frac{\partial I_D}{\partial V_{BS}} \right|_{V_{GS},V_{DS}} \)

Will deal with \( g_{mb} \) after we have dealt with body effect.
**Transconductance, $g_m$:**

\[
g_m \approx \frac{\partial I_D}{\partial V_{GS}} \bigg|_{V_{DS}, V_{BS}}
\]

- **Linear regime:**
  \[
  I_D = \frac{W}{L} \mu_e C_{ox} (V_{GS} - V_{th} - \frac{1}{2} V_{DS}) V_{DS}
  \]
  
  \[
  g_m = \frac{W}{L} \mu_e C_{ox} V_{DS}
  \]

- **Saturation regime:**
  \[
  I_{D_{sat}} = \frac{W}{2L} \mu_e C_{ox} (V_{GS} - V_{th})^2
  \]
  
  \[
  g_m = \frac{W}{L} \mu_e C_{ox} (V_{GS} - V_{th}) = \sqrt{2 \frac{W}{L} \mu_n C_{ox} I_D}
  \]
\( \square \) **Output conductance, \( g_d \):**

\[
g_d \approx \left. \frac{\partial I_D}{\partial V_{DS}} \right|_{V_{GS}, V_{BS}}
\]

- **Linear regime:**

\[
I_D = \frac{W}{L} \mu e C_{ox} (V_{GS} - V_{th} - \frac{1}{2} V_{DS}) V_{DS}
\]

\[
g_d = \frac{W}{L} \mu e C_{ox} (V_{GS} - V_{th} - V_{DS})
\]

- **Saturation regime:**

\[
I_{D_{sat}} = \frac{W}{2L} \mu e C_{ox} (V_{GS} - V_{th})^2
\]

\[
g_d = 0
\]
Add capacitors:

In saturation:

\[ C_{gdi} = 0, \quad C_{jB} = 0, \quad g_o = 0 \]
3. Short-circuit current-gain cut-off frequency

$f_T$: figure of merit to evaluate the frequency response of transistors.

$f_T$ provides insight into electron transport in MOSFET channel.

For $f_T$, MOSFET biased with output shorted from the small-signal point of view:

\[
\begin{align*}
\text{With MOSFET in saturation, } f_T \text{ defined as frequency at which small-} \\
\text{signal current gain, } h_{21} \text{ goes to unity.}
\end{align*}
\]

\[
|h_{21}(f_T)| = \left| \frac{i_d}{i_g} \right|_{v_{ds}=0} = 1
\]

As $f \downarrow \Rightarrow h_{21} \rightarrow \infty$. 
**Computation of \( f_T \): small-signal equivalent circuit model:**

\[ i_g(t) = j\omega C_{gs} v_{gs} \]
\[ i_d = g_m v_{gs} \]

\( h_{21} \) given by:

\[ h_{21} = \frac{g_m}{j\omega C_{gsi}} \]

Magnitude of \( h_{21} \) is:

\[ |h_{21}| = \frac{g_m}{\omega C_{gsi}} \]
Evolution of $h_{21}$ with frequency:

$\log |h_{21}|$ becomes unity at:

$$\omega_T = \frac{g_m}{C_{gsi}}$$

or

$$f_T = \frac{g_m}{2\pi C_{gsi}}$$
Bias dependence of $f_T$:

$$f_T = \frac{1}{2\pi} \frac{3 \mu_e (V_{GS} - V_T)}{L^2}$$

Key dependences:

- $f_T$ increases linearly with $V_{GS}$ ($g_m$ linear in $V_{GS}$)
- $f_T$ independent of $V_{DS}$ ($g_m$, $C_{gsi}$ independent of $V_{DS}$)
- $f_T$ increases linearly with $\mu_e$ ($g_m$ linear in $\mu_e$)
- $f_T$ scales as $L^{-2}$ ($g_m$ scales as $L^{-1}$, $C_{gsi}$ scales as $L$)
Physical meaning of $f_T$

$f_T$ has units of inverse time.

- Define delay time:

$$\tau_d = \frac{1}{2\pi f_T} = \frac{2}{3} \frac{L^2}{\mu_e (V_{GS} - V_T)}$$

Physical meaning of $\tau_d$:

*Delay time is average time for an electron to cross the channel from source to drain.*

- Compute delay time directly.

First, time it takes for electrons to travel distance $dy$ drifting at velocity $v_e$:

$$dt = \frac{dy}{v_e}$$

Then channel transit time is:

$$\tau_t = \int_0^L \frac{dy}{v_e(y)}$$

See Prob. 9.4 for a derivation of transit time:

$$\tau_t = \frac{2}{3} \frac{L^2}{\mu_e (V_{GS} - V_T)} = \tau_d$$

$f_T$ gives idea of intrinsic speed of transistor!
Key conclusions

• In saturation, inversion charge results in:

\[ C_{gsi} \approx \frac{2}{3} W L C_{ox} \quad \text{and} \quad C_{gdi} = 0 \]

• Transconductance. In saturation regime:

\[ g_m \propto \sqrt{I_D} \]

• Drain conductance. In saturation regime:

\[ g_d \approx 0 \]

• \( f_T \): figure of merit to characterize frequency response of MOSFET

• \( f_T \) reflects intrinsic delay associated with electron transit from source to drain