Lecture 30 - The ”Short”
Metal-Oxide-Semiconductor Field-Effect Transistor

April 23, 2007

Contents:

1. Short-channel effects

Reading assignment:

P. K. Ko, ”Approaches to Scaling.”
Key questions

- Why does it seem that in practice the drain current is significantly smaller than predicted by simple long MOSFET models?
- What is the impact of velocity saturation in the device characteristics?
1. Short-channel effects

- **Mobility degradation**: mobility dependence on $\mathcal{E}_x$ (vertical field).

Experimental observation in linear regime:

\[
\mu_{\text{eff}} = \frac{\mu_0}{1 + \left| \frac{\mathcal{E}_{av}}{\mathcal{E}_o} \right| \nu}
\]

where $\mathcal{E}_{av}$ is **average normal field in inversion layer**:

\[
\mathcal{E}_{av} = \frac{Q_{dmax} + \frac{1}{2}Q_i}{\varepsilon_s}
\]

Due to **semiconductor-oxide interface roughness**.
\( \mu_{\text{eff}} \) vs. \( E_{\text{av}} \): universal relationship for many MOSFET designs:

Parameters for the effective mobility models for electrons and holes:

<table>
<thead>
<tr>
<th></th>
<th>electrons</th>
<th>holes</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \mu_o \ (cm^2/V \cdot s) )</td>
<td>670</td>
<td>160</td>
</tr>
<tr>
<td>( E_o \ (MV/cm) )</td>
<td>0.67</td>
<td>0.7</td>
</tr>
<tr>
<td>( \nu )</td>
<td>1.6</td>
<td>1</td>
</tr>
</tbody>
</table>
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[from Arora and Richardson, VLSI Electronics: Microstructure Science, vol. 18, 1989.]
Simplified expression of $\mathcal{E}_{av}$ for n$^+$-polySi gate:

1) Relationship between $Q_{dmax}$ and $V_T$:

$$V_T = V_{FB} + \phi_{sth} - \frac{Q_{dmax}}{C_{ox}}$$

with:

$$V_{FB} = -\phi_{bi} = \frac{1}{q}(W_M-W_S) = \frac{1}{q}(W_M-\chi_s-\frac{E_g}{2}q\phi_f) = -\frac{1}{q}\frac{E_g}{2}-\phi_f$$

Plug into $V_T$ and solve for $Q_{dmax}$:

$$Q_{dmax} = -C_{ox}(V_T+\frac{E_g}{2q}+\phi_f-2\phi_f) = -C_{ox}(V_T+\frac{E_g}{2q}-\phi_f) \approx -C_{ox}V_T$$

2) Relationship between $Q_i$ and $V_{GS} - V_T$:

$$Q_i = -C_{ox}(V_{GS} - V_T)$$

3) Plug $Q_{dmax}$ and $Q_i$ into $\mathcal{E}_{av}$:

$$|\mathcal{E}_{av}| \approx \left| \frac{Q_{dmax} + \frac{1}{2}Q_i}{\epsilon_s} \right| = \frac{C_{ox}V_T + \frac{1}{2}C_{ox}(V_{GS} - V_T)}{\epsilon_s} = \frac{\epsilon_{ox}V_{GS} + V_T}{\epsilon_s2x_{ox}}$$
\[ |E_{av}| \approx \frac{\varepsilon_{ox} V_{GS} + V_T}{\varepsilon_s 2x_{ox}} \]

Key dependences:

- \( V_{GS} \uparrow \Rightarrow |E_{av}| \uparrow \Rightarrow \mu_{eff} \downarrow \)
- \( V_T \uparrow \Rightarrow |E_{av}| \uparrow \Rightarrow \mu_{eff} \downarrow \)
- \( x_{ox} \downarrow \Rightarrow |E_{av}| \uparrow \Rightarrow \mu_{eff} \downarrow \)
Several comments:

- Since $I_D \sim \mu_e$, mobility degradation more severe as $V_{GS}$ increases $\Rightarrow I_D$ won’t rise as fast with $V_{GS}$.
- Since $\mu_e$ depends on $|\varepsilon_{av}|$ $\Rightarrow \mu_{eff}$ depends on $y$. Disregard to first order $\Rightarrow$ use same $\mu_{eff}$ everywhere.
- Mobility degradation considered ”short-channel effect” because as $L \Downarrow \Rightarrow x_{ox} \Downarrow$ and $\mu$ degradation becomes important.
$g_m$ in linear regime ($V_{DS} = 0.1 \, V$) for $L_g = 1.5 \, \mu m$ MOSFET:

![Graph 1](image)

$g_m$ in linear regime ($V_{DS} = 0.1 \, V$) for $L_g = 0.18 \, \mu m$ MOSFET:

![Graph 2](image)
**Velocity saturation**

At high longitudinal fields, \( v_e \) cannot exceed \( v_{sat} \):

\[
v_e = \frac{\mu_e \mathcal{E}}{1 + \left( \frac{\mu_e \mathcal{E}}{v_{sat}} \right)^{1/n}}
\]

For inversion layer:

<table>
<thead>
<tr>
<th>( v_{sat} (cm/s) )</th>
<th>( n )</th>
<th>electrons</th>
<th>holes</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 8 \times 10^6 ) cm/s</td>
<td>2</td>
<td>( 6 \times 10^6 ) cm/s</td>
<td></td>
</tr>
<tr>
<td>( 6 \times 10^6 ) cm/s</td>
<td>1</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

To develop analytical model, use \( n = 1 \):

\[
v_e = \frac{\mu_e \mathcal{E}}{1 + \left| \frac{\mu_e \mathcal{E}}{v_{sat}} \right|}
\]
New current model:

\[ I_e = W \mu_e Q_i \frac{dV}{dy} = W \frac{\mu_e}{1 + |\frac{\epsilon}{\epsilon_{sat}}|} Q_i \frac{dV}{dy} \]

with

\[ \epsilon_{sat} = \frac{v_{sat}}{\mu_e} \]

Rewrite current equation:

\[ I_e = \left[ W \mu_e Q_i - \frac{I_e}{\epsilon_{sat}} \right] \frac{dV}{dy} \]

Plug in fundamental charge relationship:

\[ Q_i = -C_{ox}(V_{GS} - V - V_T) \]

and integrate along channel:

\[ I_eL = -W \mu_e C_{ox} \int_0^{V_{DS}} (V_{GS} - V - V_T) dV - \frac{I_e}{\epsilon_{sat}} V_{DS} \]

Solve for \( I_e \):

\[ I_e = -\frac{W \mu_e C_{ox}}{L + \frac{V_{DS}}{\epsilon_{sat}}} (V_{GS} - V_T - \frac{1}{2} V_{DS}) V_{DS} \]
Terminal drain current in linear regime:

\[ I_D = \frac{W}{L} \frac{\mu_e C_{ox}}{1 + \frac{V_{DS}}{\varepsilon_{sat} L}} (V_{GS} - V_T - \frac{1}{2} V_{DS}) V_{DS} \]

Effectively, impact of velocity saturation:

\[ \mu_e \Rightarrow \frac{\mu_e}{1 + \frac{V_{DS}}{\varepsilon_{sat} L}} \]

with \( \frac{V_{DS}}{L} \equiv \text{average longitudinal field} \).

- For \( \frac{V_{DS}}{L} \ll \varepsilon_{sat} \Rightarrow \text{velocity saturation irrelevant (mobility regime)} \).
- For \( \frac{V_{DS}}{L} \gg \varepsilon_{sat} \Rightarrow \text{velocity saturation prominent (velocity saturation regime)} \).

Since \( \varepsilon_{sat} \approx 10^5 \text{ } V/cm \) and \( V_{DS} \) order \( 1 - 10 \text{ } V \), velocity saturation important if \( L \sim 0.1 - 1 \mu m \).
Current saturation occurs when $v_{sat}$ reached anywhere in the channel ⇒ $I_D$ won’t increase anymore with $V_{DS}$:

\[
\begin{align*}
I_{Dsat} &= -Wv_eQ_i \\
&= Wv_{sat}C_{ox}(V_{GS} - V_T - V_{DSsat}) \\
&= \frac{W}{L} \frac{\mu_e C_{ox} (V_{GS} - V_T - \frac{1}{2}V_{DSsat})}{\varepsilon_{sat}L} V_{DSsat}
\end{align*}
\]

Solve for $V_{DSsat}$:

\[
V_{DSsat} = \varepsilon_{sat}L \left( \sqrt{1 + 2 \frac{V_{GS} - V_T}{\varepsilon_{sat}L}} - 1 \right)
\]
\[ V_{DS_{sat}} = \mathcal{E}_{sat} L \left( \sqrt{1 + 2 \frac{V_{GS} - V_T}{\mathcal{E}_{sat} L}} - 1 \right) \]

- For long \( L \), \( V_{DS_{sat}} \approx V_{GS} - V_T \)
- For short \( L \), \( V_{DS_{sat}} \approx \sqrt{2 \mathcal{E}_{sat} L (V_{GS} - V_T)} < V_{GS} - V_T \)

Velocity saturation results in *premature current saturation* and less current:

\[ I_{D_{sat}} = W v_{sat} C_{ox} (V_{GS} - V_T - V_{DS_{sat}}) \]
Experiments:

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Current-voltage characteristics:
Impact of velocity saturation on transconductance:

\[ g_m = \frac{\partial I_{D_{sat}}}{\partial V_{GS}} = W v_{sat} C_{ox} \left( 1 - \frac{\partial V_{D_{Sat}}}{\partial V_{GS}} \right) \]

with

\[ \frac{\partial V_{D_{Sat}}}{\partial V_{GS}} = \frac{1}{\sqrt{1 + \frac{2(V_{GS} - V_T)}{\varepsilon_{sat} L}}} \]

Then:

\[ g_m = W v_{sat} C_{ox} \left( 1 - \frac{1}{\sqrt{1 + \frac{2(V_{GS} - V_T)}{\varepsilon_{sat} L}}} \right) \]

In the limit of short \( L \):

\[ g_m = W v_{sat} C_{ox} \]

In the limit of short \( L \), \( g_m \) determined only by \( x_{ox} \).
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MOSFET $I_d$ scaling (Ko, 1989)

- Long-channel theory
- Mobility degradation due to $E_x$
- Velocity degradation due to $E_y$
- Short-channel theory

$I_d$ at $V_{g} - V_t = 5$ V ($\mu$A/$\mu$m)

$L_g$ ($\mu$m)

Cite as: Jesús del Alamo, course materials for 6.720J Integrated Microelectronic Devices, Spring 2007. MIT OpenCourseWare (http://ocw.mit.edu/), Massachusetts Institute of Technology. Downloaded on [DD Month YYYY].
Key conclusions

• Inversion layer mobility degraded by transversal field due to roughness of semiconductor-oxide interface ⇒ \( I_D \) lower than predicted by simple models.

• There is a *universal relationship between mobility and average transversal field in inversion layer*.

• For short \( L \), velocity saturation in inversion layer important ⇒ \( I_D \) lower than predicted by simple models.

• Velocity saturation ⇒ premature MOSFET saturation ⇒ \( V_{DS_{sat}} \) lower than predicted by simple models.

• Velocity saturation ⇒ \( g_m \) saturation in limit of short \( L \):

\[
g_m = W v_{sat} C_{ox}
\]