Lecture 36 - Bipolar Junction Transistor
(cont.)

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1. Current-voltage characteristics of ideal BJT (cont.)
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3. Small-signal behavior of ideal BJT

Reading material:

del Alamo, Ch. 11, §§11.2 (11.2.5), 11.3, 11.4 (11.4.1)
Key questions

• How do the output characteristics of the ideal BJT look like?

• How do the charge-voltage characteristics of the ideal BJT look like?

• What is the topology of the small-signal equivalent circuit model of the ideal BJT in FAR?

• What are the key dependencies of its elements?
1. Current-voltage characteristics of ideal BJT (cont.)

Ideal BJT current equations (superposition of forward active + reverse):

\[
I_C = I_S \left( \exp \frac{qV_{BE}}{kT} - \exp \frac{qV_{BC}}{kT} \right) - \frac{I_S}{\beta_R} \left( \exp \frac{qV_{BC}}{kT} - 1 \right)
\]

\[
I_B = \frac{I_S}{\beta_F} \left( \exp \frac{qV_{BE}}{kT} - 1 \right) + \frac{I_S}{\beta_R} \left( \exp \frac{qV_{BC}}{kT} - 1 \right)
\]

\[
I_E = -\frac{I_S}{\beta_F} \left( \exp \frac{qV_{BE}}{kT} - 1 \right) - I_S \left( \exp \frac{qV_{BE}}{kT} - \exp \frac{qV_{BC}}{kT} \right)
\]

Equivalent circuit model representation:

![Equivalent circuit model](image)

Complete model has only three parameters: \( I_S, \beta_F, \) and \( \beta_R. \)
Common-emitter output I-V characteristics

**vs. $V_{CB}$:**

\[
V_{CE,\text{sat}} = -V_{BCon} + V_{BEon}
\]
$I_C$ vs. $V_{CB}$ with $I_B$ as parameter:
Where is the reverse regime?
Common-emitter output characteristics:
Zoom into inverse regime:
2. Charge-voltage characteristics of ideal BJT

In BJT, two types of stored charge:

- depletion layer charge
- minority carrier charge

In forward-active regime:
\square \textbf{Depletion layer charge}

In B-E and B-C SCR’s, respectively:

\[ Q_{jE} = A_E \sqrt{\frac{2\epsilon q N_E N_B (\phi_{biE} - V_{BE})}{N_E + N_B}} \]

\[ Q_{jC} = A_C \sqrt{\frac{2\epsilon q N_B N_C (\phi_{biC} - V_{BC})}{N_B + N_C}} \]

\(\phi_{biE}\) and \(\phi_{biC}\) are respective built-in potentials.

Since \(N_E \gg N_B \gg N_C\),

\[ Q_{jE} \approx A_E \sqrt{2\epsilon q N_B (\phi_{biE} - V_{BE})} \]

\[ Q_{jC} \approx A_C \sqrt{2\epsilon q N_C (\phi_{biC} - V_{BC})} \]

Depletion capacitance:

\[ C_{je} = \frac{\partial Q_{jE}}{\partial V_{BE}} \approx A_E \sqrt{\frac{\epsilon q N_B}{2(\phi_{biE} - V_{BE})}} = \frac{C_{jEO}}{\sqrt{1 - \frac{V_{BE}}{\phi_{biE}}}} \]

\[ C_{jc} = \frac{\partial Q_{jC}}{\partial V_{BC}} \approx A_C \sqrt{\frac{\epsilon q N_C}{2(\phi_{biC} - V_{BC})}} = \frac{C_{jCO}}{\sqrt{1 - \frac{V_{BC}}{\phi_{biC}}}} \]
Minority carrier charge

Excess minority carriers in QNR’s ⇒ excess majority carriers to keep quasi-neutrality ⇒ diffusion capacitance.

Key result from pn diode: in ”short” or ”transparent” QNR:

\[
\text{stored charge} = \text{minority carrier transit time} \times \text{injected minority carrier current}
\]

For emitter in FAR:

\[
Q_E = \tau_{tE} I_B
\]

with hole transit time:

\[
\tau_{tE} = \frac{W_E^2}{2D_E}
\]
• For base in FAR:

\[
Q_B = \tau_{tB} I_C
\]

with electron transit time:

\[
\tau_{tB} = \frac{W_B^2}{2D_B}
\]
Comments:

- Units of $Q_E$ and $Q_B$ are C.
- $Q_E$ and $Q_B$ scale with $A_E$.

Total minority carrier charge in FAR:

$$Q_F = Q_E + Q_B = \tau_{tE} I_B + \tau_{tB} I_C = \left(\frac{\tau_{tE}}{\beta_F} + \tau_{tB}\right) I_C = \tau_F I_C$$

$$\tau_F \equiv \text{intrinsic delay [s]}$$

$\tau_F$ is overall time constant for minority carrier storage in BJT in FAR:

$$\tau_F = \frac{\tau_{tE}}{\beta_F} + \tau_{tB}$$

Note: emitter contribution to $\tau_F$ is $\tau_{tE}/\beta_F$ because $I_B$ is $\beta_F$ times smaller than $I_C$.

If $V_{BE}$ changes, $Q_E$ and $Q_B$ change $\Rightarrow$ capacitive effect:

$$C_F = \frac{dQ_F}{dV_{BE}} = \tau_F \frac{qI_C}{kT}$$
Location of this capacitance? Think of which terminals supply stored charge (minority and majority carriers):

For $Q_E$:

- minority carriers (holes) injected from base
- majority carriers (electrons) come from emitter contact

For $Q_B$:

- minority carriers (electrons) injected from emitter
- majority carriers (holes) come from base contact

Equivalent-circuit model:
Similar picture in reverse regime: charge storage in base and collector

\[ Q_R = \tau_R I_E \]

\( \tau_R \) a bit complicated because it accounts for charge storage in intrinsic and extrinsic base and collector regions.

Diffusion capacitance:

\[ C_R = \frac{dQ_R}{dV_{BC}} = \tau_R \frac{qI_E}{kT} \]

Located between base and collector terminals.
By superposition, complete equivalent circuit model valid in all four regimes:

\[ I_S (\exp \frac{qV_{BE}}{kT} - \exp \frac{-qV_{BC}}{kT}) \]
3. Small-signal behavior of ideal BJT

In analog (and digital) applications, interest in behavior of BJT to small-signal applied on top of bias
⇒ small-signal equivalent circuit model.

☐ Small-signal equivalent circuit model in FAR

Must linearize hybrid-π model in FAR:

-Non-linear voltage-controlled current source linearized to linear voltage-controlled current source.

-Diode linearized to resistor.

-Charge storage elements linearized to capacitors.
• **Linearized voltage-controlled current source**

Apply small signal $v_{be}$ on top of bias $V_{BE}$.

Collector current:

$$I_C + i_C = I_S \exp \left( \frac{q(V_{BE} + v_{be})}{kT} \right) \approx I_S \exp \left( \frac{qV_{BE}}{kT} \right) \left( 1 + \frac{qv_{be}}{kT} \right) = I_C (1 + \frac{qv_{be}}{kT})$$

Small-signal collector current:

$$i_C = \frac{qI_C}{kT} v_{be}$$

Define **transconductance**: 

$$g_m = \frac{qI_C}{kT}$$

$g_m$ depends only on absolute value of $I_C$ and $T$ (unlike MOSFET, where $g_m$ depends on device geometry)
• Linearized diode

\[ I_B + i_b = I_S \exp \left( \frac{q(V_{BE} + v_{be})}{kT} \right) \approx I_S \frac{q}{\beta_F} \exp \left( \frac{V_{BE}}{kT} \right) \left( 1 + \frac{q v_{be}}{kT} \right) \]

Base current:

\[ I_B + i_b = I_S \exp \left( \frac{q(V_{BE} + v_{be})}{kT} \right) \approx I_S \frac{q}{\beta_F} \exp \left( \frac{V_{BE}}{kT} \right) \left( 1 + \frac{q v_{be}}{kT} \right) \]

Small-signal base current:

\[ i_b = \frac{qI_B}{kT} v_{be} \]

Define conductance:

\[ g_\pi = \frac{qI_B}{kT} = \frac{q}{kT} \frac{I_C}{\beta_F} = g_m \frac{1}{\beta_F} \]

Then, in general,

\[ g_\pi \ll g_m \]
• Capacitors

\[ Q_{JE} \rightarrow C_{je} \]

\[ Q_{JC} \rightarrow C_{jc} \]

\[ Q_F \rightarrow C_\pi = \tau_F \frac{qI_C}{kT} \]

Two components in \( C_\pi \):

Note:

\[ C_\pi = \tau_F g_m \]
• Small-signal equivalent circuit model for ideal BJR in FAR:

![Circuit Diagram]

Cite as: Jesús del Alamo, course materials for 6.720J Integrated Microelectronic Devices, Spring 2007. MIT OpenCourseWare (http://ocw.mit.edu/), Massachusetts Institute of Technology. Downloaded on [DD Month YYYY].
Key conclusions

- In BJT, two types of stored charge: depletion layer charge and minority carrier charge.

- Depletion layer charge accounted through depletion capacitances.

- Minority carrier charge accounted through time constant $\tau_F$ (intrinsic delay):

$$\tau_F = \frac{\tau_{tE}}{\beta_F} + \tau_{tB}$$

Emitter contribution to $\tau_F$ is $\beta_F$ times smaller than $\tau_{tE}$ because $I_B$ is $\beta_F$ times smaller than $I_C$.

- Non-linear hybrid-$\pi$ model for ideal BJT including charge storage elements:
- Small-signal equivalent circuit model of ideal BJT in FAR:

\[ \begin{align*}
    g_m &= \frac{qI_C}{kT} \\
    g_\pi &= \frac{qI_B}{kT} = \frac{g_m}{\beta_F} \\
    C_\pi' &= \tau_F g_m
\end{align*} \]