Lecture 37 - Bipolar Junction Transistor

(cont.)

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Contents:

1. Common-emitter short-circuit current-gain cut-off frequency, $f_T$

Reading material:

del Alamo, Ch. 11, §11.4.2
Key questions

• How is the frequency response of a transistor assessed?
• What determines the frequency response of an ideal BJT?
• How can the frequency response of a BJT be engineered?
1. Common-emitter short-circuit current-gain cut-off frequency, $f_T$

$f_T$: high-frequency figure of merit for transistors

Short-circuit means from the small-signal point of view.

BJT is biased in FAR.

Focus on small-signal current gain:

$$h_{21} = \left. \frac{i_c}{i_b} \right|_{v_{ce}=0}$$

For low frequency, $h_{21} \rightarrow \beta_F$, for high frequency $h_{21}$ rolls off due to capacitors.

Definition of $f_T$: frequency at which $|h_{21}| = 1$. 
Small-signal equivalent circuit model:

\[ i_c = g_m v_{be} - j\omega C_{jc} \]
\[ i_b = [g_\pi + j\omega (C_\pi + C_{je} + C_{jc})] v_{be} \]

Then:

\[ h_{21} = \frac{g_m - j\omega C_{jc}}{g_\pi + j\omega (C_\pi + C_{je} + C_{jc})} \]

Magnitude of \( h_{21} \):

\[ |h_{21}| = \frac{\sqrt{g_m^2 + \omega^2 C_{jc}^2}}{\sqrt{g_\pi^2 + \omega^2 (C_\pi + C_{je} + C_{jc})^2}} \]
\[ |h_{21}| = \frac{\sqrt{g_m^2 + \omega^2 C_{jc}^2}}{\sqrt{g_{\pi}^2 + \omega^2 (C_{\pi} + C_{je} + C_{jc})^2}} \]

Bode plot of \(|h_{21}|\):

\[
\begin{aligned}
\log |h_{21}| &\sim \frac{g_m}{g_{\pi}} = \beta_F \\
\log \omega &\sim -1 \\
\omega &\sim \omega_{\beta}, \omega_{\beta}, \omega_{c}
\end{aligned}
\]

Three regimes in \(|h_{21}|\):

- low frequency, \(\omega \ll \omega_{\beta}\):
  \[ |h_{21}| \sim \frac{g_m}{g_{\pi}} = \beta_F \]

- intermediate frequency, \(\omega_{\beta} \ll \omega \ll \omega_{c}\):
  \[ |h_{21}| \sim \frac{g_m}{\omega(C_{\pi} + C_{je} + C_{jc})} \]

- high frequency, \(\omega \gg \omega_{c}\):
  \[ |h_{21}| \sim \frac{C_{jc}}{C_{\pi} + C_{je} + C_{jc}} \]
Angular frequencies that separate three regimes:

\[
\omega_\beta = \frac{g_\pi}{C_\pi + C_{je} + C_{jc}}
\]

\[
\omega_c = \frac{g_m}{C_{jc}}
\]

Angular frequency at which \(|h_{21}| = 1\):

\[
\omega_T = \frac{g_m}{C_\pi + C_{je} + C_{jc}}
\]

In terms of frequency:

\[
f_T = \frac{g_m}{2\pi(C_\pi + C_{je} + C_{jc})}
\]

Note:

\[
\omega_\beta = \frac{\omega_T}{\beta_F}
\]
Physical meaning of \( f_T \)

\( 1/2\pi f_T \) has units of time. Define delay time:

\[
\tau_d = \frac{1}{2\pi f_T} = \frac{C\pi}{g_m} + \frac{C_{je}}{g_m} + \frac{C_{jc}}{g_m} = \tau_{tB} + \frac{\tau_{tE}}{\beta_F} + \frac{C_{je}}{g_m} + \frac{C_{jc}}{g_m}
\]

Four delay components in \( \tau_d \).

Consider response of BJT to a step-input base current:

At \( t = 0 \)

\[
I_B \rightarrow I_B + i_b
\]

As \( t \to \infty \)

\[
V_{BE} \rightarrow V_{BE} + v_{be}
\]

\[
I_C \rightarrow I_C + i_c = I_C + \beta_F i_b.
\]
How much time does it take for \( i_C \) to reach its final value?

Charge must be delivered to four regions in BJT:

- **Quasi-neutral emitter**
  \[
  q_e = \tau_{tE} i_b
  \]

- **Quasi-neutral base**
  \[
  q_b = \tau_{tB} i_c
  \]

- **Emitter-base depletion region**
  \[
  q_{je} = C_{je} v_{be} = \frac{C_{je}}{g_m} i_c
  \]

- **Base-collector depletion region**
  \[
  q_{jc} = C_{jc} v_{bc} = C_{jc} v_{be} = \frac{C_{jc}}{g_m} i_c
  \]

Charge delivered at constant rate to base. Time that it takes for all charge to be delivered:

\[
\tau_\beta = \frac{q_e + q_b + q_{je} + q_{jc}}{i_b} = \tau_{tE} + \beta F (\tau_{tB} + \frac{C_{je}}{g_m} + \frac{C_{jc}}{g_m}) = \frac{1}{2\pi f_\beta}
\]
How much time does it take for $i_C$ to build up to $I_C + i_b$?

Since $i_c = \beta F i_b$,

$$
\tau_d = \frac{\tau_\beta}{\beta_F} = \frac{\tau_{te}}{\beta_F} + \tau_{tB} + \frac{C_{je}}{g_m} + \frac{C_{jc}}{g_m} = \frac{1}{2\pi f_T}
$$

- $\tau_d = \frac{1}{2\pi f_T}$: delay time before $i_C$ increases to $I_C + i_b$
- $\tau_\beta = \frac{1}{2\pi f_\beta}$: delay time before $i_C$ increases to $I_C + \beta F i_b$

With sinusoidal input:

\[ f \uparrow \Rightarrow \text{fraction of } i_b \text{ that goes into capacitors} \uparrow \Rightarrow v_{be} \downarrow \Rightarrow i_c \downarrow. \]

At $f_T$: $|i_c| = |i_b|$
Key dependencies of $f_T$ in ideal BJT

$\star$ $f_T$ dependence on $I_C$:

Rewrite $f_T$:

$$f_T = \frac{g_m}{2\pi(C_\pi + C_{je} + C_{jc})} = \frac{1}{2\pi \tau_F} \left( 1 + \frac{kT}{q \tau_F} \frac{C_{je} + C_{jc}}{I_C} \right)$$

Two limits:

- Small $I_C$: limited by depletion capacitances

  $$f_T \simeq \frac{q}{2\pi kT} \frac{I_C}{C_{je} + C_{jc}}$$

- Large $I_C$: limited by intrinsic delay (dominated by $\tau_{tB}$)

  $$f_T \simeq \frac{1}{2\pi \tau_F}$$
Alternative view of $I_C$ dependence:

\[ \tau_d = \frac{\tau_{tE}}{\beta_F} + \tau_{tB} + \frac{C_{je}}{g_m} + \frac{C_{jc}}{g_m} = \frac{1}{2\pi f_T} \]

Standard experimental technique to extract $\tau_F$ and $C_{je} + C_{jc}$:
\* $f_T$ dependence on $V_{BC}$:

$V_{CB} \uparrow$ (B-C junction is more reverse biased) $\Rightarrow C_{je} \downarrow \Rightarrow f_T \uparrow$

[but only in low $I_C$ regime of $f_T$]

$\log f_T$

$\frac{1}{2\pi \tau_F}$

$V_{CB} \uparrow$

$log I_C$

\* $f_T$ dependence on device layout:

- For low $I_C$: $f_T$ dominated by $C_{je}, C_{jc}$

\[
\frac{C_{je}}{g_m} \propto \frac{A_E C_{jeo}}{I_C}
\]

\[
\frac{C_{jc}}{g_m} \propto \frac{A_C C_{jco}}{I_C}
\]

If $A_E \uparrow$ or $A_C \uparrow$ (keeping $I_C$ constant) $\Rightarrow f_T \downarrow$

- For high $I_C$: $f_T$ dominated by $\tau_F$; $f_T$ independent of $A_E$ or $A_C$
Device design strategies for improving $f_T$

Four delay terms in $f_T$:

$$\tau_d = \frac{1}{2\pi f_T} = \frac{\tau_{tE}}{\beta_F} + \tau_{tB} + \frac{C_{je}}{g_m} + \frac{C_{jc}}{g_m}$$

Strategies to reduce each delay component:

* Emitter charging time, $\frac{\tau_{tE}}{\beta_F}$, minimized by
  
  - enhancing $\beta_F$,
  - having a shallow emitter ($\tau_{tE} \sim W_E^2$),
  - building steep doping profile in emitter.

$\frac{\tau_{tE}}{\beta_F}$ small contribution to $\tau_d$, not much payoff.
* Base transit time, $\tau_{tB}$, minimized by

- reducing $W_B$ ($\tau_{tB} \sim W_B^2$),
- introducing drift field in base (through impurity gradient or SiGe composition gradient).

Significant device engineering towards minimizing $\tau_{tB}$.

Example 1 [Kasper 1993]:

![Graph showing transit frequency $f_T$ versus SiGe thickness (effective base width)]

**Fig. 2**: Transit frequency $f_T$ versus SiGe thickness (effective base width)

Example 2 [Yamazaki, IEDM 1990, p. 309]:

Example 3 [Crabbé, IEDM 1990, p. 17]:

![Graph showing the collector current dependence of $f_T$ at 298K and 85K for Si and SiGe devices. In both cases, the peak $f_T$ increases at lower temperature as well as the associated collector current.]

* E-B SCR charging time, $C_{je}/g_m$:

$$\frac{C_{je}}{g_m} \propto \frac{A_E C_{jeo}}{I_C} = \frac{C_{jeo}}{J_C}$$

Minimized by:

- $N_B \downarrow$
- tailoring doping profiles at E-B junction

* B-C SCR charging time, $C_{jc}/g_m$:

$$\frac{C_{jc}}{g_m} \propto \frac{A_C C_{jco}}{I_C} = \frac{A_C C_{jco}}{A_E J_C}$$

Minimized by:

- $N_C \downarrow$
- tailoring doping profiles at B-C junction.
- tightening layout of transistor: $\frac{A_C}{A_E} \rightarrow 1$
Key conclusions

- $f_T$: high-frequency figure of merit for transistors: frequency at which $|h_{21}| = 1$.

- $f_T$ of ideal BJT:

$$f_T = \frac{g_m}{2\pi(C_\pi + C_{je} + C_{jc})}$$

- Delay time, $\tau_d = \frac{1}{2\pi f_T}$: time it takes for step increase in $i_B$ to yield an identical step increase in $i_C$.

- Most effective ways to engineer $f_T$:
  - reduce $W_B$
  - introduce drift field in base (through impurity gradient or SiGe composition gradient)
  - tighten layout: $\frac{A_C}{A_E} \rightarrow 1$