Lecture 9 - Carrier Flow (cont.)

February 23, 2007

Contents:

1. Shockley’s Equations
2. Simplifications of Shockley equations to 1D quasi-neutral situations
3. Majority-carrier type situations

Reading assignment:

del Alamo, Ch. 5, §§5.3-5.5

Quote of the day:

"If in discussing a semiconductor problem, you cannot draw an energy band diagram, then you don’t know what you are talking about.”

Key questions

- How can the equation set that describes carrier flow in semiconductors be simplified?
- In regions where carrier concentrations are high enough, quasi-neutrality holds in equilibrium. How about out of equilibrium?
- What characterizes majority-carrier type situations?
1. Shockley’s Equations

<table>
<thead>
<tr>
<th>Equation</th>
<th>Expression</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gauss’ law:</td>
<td>$\vec{\nabla} \cdot \vec{E} = \frac{q}{\epsilon} (p - n + N_D^+ - N_A^-)$</td>
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<td>Electron current equation:</td>
<td>$\vec{J}_e = -qn\vec{v}_e^{drift} + qD_e \vec{\nabla} n$</td>
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<td>Hole current equation:</td>
<td>$\vec{J}_h = qp\vec{v}_h^{drift} - qD_h \vec{\nabla} p$</td>
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<td>Electron continuity equation:</td>
<td>$\frac{\partial n}{\partial t} = G_{ext} - U(n, p) + \frac{1}{q} \vec{\nabla} \cdot \vec{J}_e$</td>
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<tr>
<td>Hole continuity equation:</td>
<td>$\frac{\partial p}{\partial t} = G_{ext} - U(n, p) - \frac{1}{q} \vec{\nabla} \cdot \vec{J}_h$</td>
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<tr>
<td>Total current equation:</td>
<td>$\vec{J}_t = \vec{J}_e + \vec{J}_h$</td>
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System of non-linear, coupled partial differential equations.
2. Simplifications of Shockley equations to 1D quasi-neutral situations

- **One-dimensional approximation**

In many cases, complex problems can be broken into several 1D subproblems.

Example: integrated p-n diode

\[
\nabla \Rightarrow \frac{\partial}{\partial x}
\]

1D approximation: \( \nabla \Rightarrow \frac{\partial}{\partial x} \)
Shockley’s equations in 1D:

\[
\begin{align*}
\text{Gauss’ law:} & \quad \frac{\partial \mathcal{E}}{\partial x} = \frac{q}{\epsilon} (p - n + N_D - N_A) \\
\text{Electron current equation:} & \quad J_e = -qv_e^{drift}(\mathcal{E}) + qD_e \frac{\partial n}{\partial x} \\
\text{Hole current equation:} & \quad J_h = qpv_h^{drift}(\mathcal{E}) - qD_h \frac{\partial p}{\partial x} \\
\text{Electron continuity equation:} & \quad \frac{\partial n}{\partial t} = G_{ext} - U(n, p) + \frac{1}{q} \frac{\partial J_e}{\partial x} \\
\text{Hole continuity equation:} & \quad \frac{\partial p}{\partial t} = G_{ext} - U(n, p) - \frac{1}{q} \frac{\partial J_h}{\partial x} \\
\text{Total current equation:} & \quad J_t = J_e + J_h
\end{align*}
\]

Equation set difficult because of coupling through Gauss’ law.

Two broad classes of important situations break Gauss’law coupling:

1. Carrier concentrations are high: quasi-neutral situation:

\[\rho \approx 0 \Rightarrow \frac{\partial \mathcal{E}}{\partial x} \approx 0\]

2. Carrier concentrations are very low: space-charge and high-resistivity situations:

\[\mathcal{E} \text{ independent of } n, p\]
Overview of simplified carrier flow formulations

General drift-diffusion situation
(Shockley's equations)

1D approx.

Quasi-neutral situation
(negligible volume charge)

Space-charge situation
(field independent of n, p)

Majority-carrier type situation
(V≠0, n'=p'=0)

Minority-carrier type situation
(V=0, n'=p'≠0, LLI)
Each formulation uniquely applies to a different region in a device.

Example: npn BJT in forward-active regime
Quasi-neutral approximation

At every location, the net volume charge that arises from a discrepancy of the concentration of positive and negative species is negligible in the scale of the charge density that is present.

QN approximation eliminates Gauss’ law from the set:

\[ \rho = q(p - n + N_D^+ - N_A^-) = q(p_o - n_o + N_D^+ - N_A^-) + q(p' - n') \]

- Quasi-neutrality in equilibrium:

\[ \left| \frac{p_o - n_o + N_D^+ - N_A^-}{N_D^+ - N_A^-} \right| \ll 1 \]

which implies

\[ n_o - p_o \simeq N_D^+ - N_A^- \]

- Additionally, quasi-neutrality outside equilibrium:

\[ \left| \frac{p' - n'}{n'} \right| \simeq \left| \frac{p' - n'}{p'} \right| \ll 1 \]

which implies:

\[ p' \simeq n' \]

- QN approximation good if \( n, p \) high \( \Rightarrow \) carriers move to erase \( \rho \).

- QN holds if length scale of problem \( \gg \) Debye length
\begin{itemize}
\item Consequence of quasi-neutrality
\end{itemize}

1. Uncouple Gauss’ law from rest of system:

\[ \rho \approx 0 \Rightarrow \frac{\partial \mathcal{E}}{\partial x} \approx 0 \]

If, in general define:

\[ \mathcal{E} = \mathcal{E}_o + \mathcal{E}' \]

Then, in equilibrium:

\[ \frac{\partial \mathcal{E}_o}{\partial x} = \frac{q}{\epsilon} (p_o - n_o + N_D^+ - N_A^-) \]

and out of equilibrium:

\[ \frac{\partial \mathcal{E}'}{\partial x} = \frac{q}{\epsilon} (p' - n') \]

\( \mathcal{E}_o \) computed as in Ch. 4. Here will learn to compute \( \mathcal{E}' \).
2. Subtract one continuity equation from the other:

\[
\frac{\partial J_t}{\partial x} = q \frac{\partial (n - p)}{\partial t} = -\frac{\partial \rho}{\partial t}
\]

*continuity equation for net volume charge*: if \( J_t \) changes with position, \( \rho \) changes with time.

Easier to see in integral form:

\[
\int_S J_t \cdot d\vec{S} = -\frac{\partial}{\partial t} \int_V \rho \, dV
\]

- In *Static case*:
  \[
  \frac{\partial \rho}{\partial t} = 0 \Rightarrow \frac{\partial J_t}{\partial x} = 0, \quad J_t \text{ independent of } x
  \]

- In *Dynamic case*, we also have in most useful situations:
  \[
  \frac{\partial \rho}{\partial t} \approx 0 \text{ in times scale of interest}
  \]

[will discuss soon]
Simplified set of Shockley equations for 1D quasi-neutral situations

\[ p - n + N_D - N_A \simeq 0 \]

\[ J_e = -qn v_e^{drift} + qD_e \frac{\partial n}{\partial x} \]

\[ J_h = qp v_h^{drift} - qD_h \frac{\partial p}{\partial x} \]

\[ \frac{\partial n}{\partial t} = G_{ext} - U + \frac{1}{q} \frac{\partial J_e}{\partial x} \quad \text{or} \quad \frac{\partial p}{\partial t} = G_{ext} - U - \frac{1}{q} \frac{\partial J_h}{\partial x} \]

\[ \frac{\partial J_t}{\partial x} \simeq 0 \]

\[ J_t = J_e + J_h \]
3. Majority-carrier type situations

Voltage applied to extrinsic quasi-neutral semiconductor without upsetting the equilibrium carrier concentrations.

☐ Remember what a battery does:

- Battery picks up electrons from positive terminal, increases their potential energy and puts them at the negative terminal.
- If provided with a path (resistor), electrons flow.
Characteristics of majority carrier-type situations:

- electric field imposed from outside
- electrons and holes drift
- electron and hole concentrations unperturbed from TE

Simplifications:

- neglect contribution of minority carriers
- neglect time derivatives of carrier concentrations

⇒ problem becomes completely quasi-static
Simplification of majority carrier current (n-type):

Must distinguish between internal field in TE ($\mathcal{E}_o$) and total field outside equilibrium ($\mathcal{E}$).

For simplicity, do in low-field limit (exact case done in notes).

In equilibrium:

\[ J_{eo} = q\mu_e n_o \mathcal{E}_o + qD_e \frac{dn_o}{dx} = 0 \]

Out of equilibrium:

\[ J_e \approx q\mu_e n_o \mathcal{E} + qD_e \frac{dn_o}{dx} \]

Hence:

\[ J_e = q\mu_e n_o (\mathcal{E} - \mathcal{E}_o) = q\mu_e n_o \mathcal{E}' \]

In the more general case (see notes):

\[ J_e = -qn_o [v_{de}(\mathcal{E}) - v_{de}(\mathcal{E}_o)] \]
Equation set for 1D majority-carrier type situations:

<table>
<thead>
<tr>
<th>n-type</th>
<th>p-type</th>
</tr>
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<tbody>
<tr>
<td>( n \simeq n_o \simeq N_D )</td>
<td>( p \simeq p_o \simeq N_A )</td>
</tr>
<tr>
<td>( J_e = -qn_o[v_{de}(E) - v_{de}(E_o)] )</td>
<td>( J_h = qp_o[v_{dh}(E) - v_{dh}(E_o)] )</td>
</tr>
<tr>
<td>( \frac{dJ_e}{dx} \simeq 0, \frac{dJ_h}{dx} \simeq 0, \frac{dJ_t}{dx} \simeq 0 )</td>
<td></td>
</tr>
<tr>
<td>( J_t \simeq J_e )</td>
<td>( J_t \simeq J_h )</td>
</tr>
</tbody>
</table>
Example 1: *Integrated Resistor* with uniform doping (n-type)

Uniform doping $\Rightarrow \mathcal{E}_o = 0$, then:

$$J_t = -qN_D \mu_e \varepsilon^{drift}(\mathcal{E})$$

- If $\mathcal{E}$ not too high,

$$J_t \simeq qN_D \mu_e \mathcal{E}$$

I-V characteristics:

$$I = WtqN_D \mu_e \frac{V}{L}$$
• In general (low and high fields):

\[ I = W t q N_D \frac{v_{sat}}{1 + \frac{v_{sat}}{\mu_e} \frac{L}{V}} \]

which for high fields saturates to:

\[ I_{sat} = W t q N_D v_{sat} \]
Key conclusions

- **Shockley equations**: system of equations that describes carrier phenomena in semiconductors in the drift-diffusion regime.

- **Quasi-neutral** approximation appropriate if semiconductor is sufficiently extrinsic: \( \rho \simeq 0 \Rightarrow \)

  \[
  n_o - p_o \simeq N_D - N_A \quad n' \simeq p'
  \]

- Consequence of quasi-neutrality:

  \[
  J_t \neq J_t(x)
  \]

- **Majority carrier-type situations** characterized by application of external voltage without perturbing carrier concentrations.

- Majority-carrier type situations dominated by drift of majority carriers.

- Integrated resistor:

  - for low voltages, current proportional to voltage across
  - for high voltages, current saturates due to \( v_{sat} \)
Self-study

- Integral form of continuity equations and consequences.
- Exercises 5.1, 5.2.
- Non-uniformly doped resistor
- Sheet resistance