Quiz 1

Quiz Out: 10/16/06  Quiz Due: 10/18/06 at the beginning of class

NAME:

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Problem 1 (35 points)

Consider an electron that is incident from the left on the step barrier \( V(x) \) where

\[
V(x) = \begin{cases} 
0 & \text{if } x < 0 \\
V_o & \text{if } x \geq 0 
\end{cases}
\]

The eigenfunction at a constant energy \( E \) can be written as

\[
\psi(x) = \begin{cases} 
e^{ik_1x} + Be^{-ik_1x} & \text{if } x \leq 0 \\
Ce^{ik_2x} & \text{if } x \geq 0
\end{cases}
\]

(a) Explain why the wavefunction \( \psi(x) \) has the above form. What are the values of \( k_1 \) and \( k_2 \) in terms of \( E, V_o \) and fundamental constants?

(b) Find the transmitted probability current density, \( J_{\text{trans}}(x) \) in terms of \( B, C, k_1, k_2 \) and fundamental constants.

(c) Find the values of \( B \) and \( C \) in terms of \( k_1 \) and \( k_2 \).

(d) Find the reflection \( R \) and transmission \( T \) coefficients. (Note you can check that you have the correct answer with the result on page 4 of the formula sheet.)
Problem 2 (35 points)
Consider the simple harmonic oscillator for a particle with mass $m$ and oscillation frequency $\omega_0$. The energy eigenstates are given by the set $\phi_n(x)$ which have eigenenergies $E_n = \hbar \omega_0 (n + 1/2)$.
You are given a wavefunction whose initial state in time is
$$\Psi(x, t = 0) = c_0 \phi_0 + c_3 \phi_3$$
(a) What is the wavefunction $\Psi(x, t)$ for all time?
(b) What is the probability density of finding the particle at some position $x_1$ as a function of time?
(c) What is the expectation value of the Energy $\langle E \rangle$?
(d) Find $\langle x \rangle$, $\langle x^2 \rangle$, and $\langle x^3 \rangle$ in terms of $c_0$, $c_3$ and the constants of the system.
Problem 3 (30 points)

The Hamiltonian $H$ is a function of momentum $p$ and is given by

$$H = \frac{p^2}{2m} + \alpha p$$

where $m$ is the mass of the particle and $\alpha$ is a constant with units of velocity.

(a) Use Ehrenfest’s Theorem to find

$$\frac{d}{dt} \langle x \rangle \quad \text{and} \quad \frac{d}{dt} \langle p \rangle.$$

(b) Write down Schoedinger’s Equation in $x$-space for this Hamiltonian.

(c) Find the eigenfunctions and eigenenergies.