QC1: Quantum Computing with Superconductors

1. Introduction to Quantum Computation
   1. The Unparalleled Power of a Quantum Computer
   2. Two state systems: qubits
   3. Types of Qubits

2. Quantum Circuits

3. Building a Quantum Computer with Superconductors
   1. Quantizing Superconducting Josephson Circuits
   2. Dynamics of Two-Level Quantum Systems
   3. Types of superconducting qubits
   4. Experiments on superconducting qubits
      1. Charge qubits
      2. Phase/Flux qubits
      3. Hybrid qubits
      4. Advantages of superconductors as qubits

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Quantum Computing

Qubits are two level systems
a) Spin states can be true two level systems, or
b) Any two quantum energy levels can also be used

We will call the lower energy state $|0\rangle$ and the higher energy state $|1\rangle$

In general, the wave function can be in a superposition of these two states

$$\psi = a|0\rangle + b|1\rangle$$
Computing with Quantum States

- Consider two qubits, each in superposition states
  \[ |\psi_A\rangle = |0\rangle_A + |1\rangle_A \quad |\psi_B\rangle = |0\rangle_B + |1\rangle_B \]

- We can re-write these states a single state of the 2-e system
  \[ |\psi\rangle = |\psi_A\rangle |\psi_B\rangle = (|0\rangle_A + |1\rangle_A) \otimes (|0\rangle_B + |1\rangle_B) \]
  \[ = |0\rangle_A |0\rangle_B + |0\rangle_A |1\rangle_B + |1\rangle_A |0\rangle_B + |1\rangle_A |1\rangle_B \]

- All four “numbers” exist simultaneously
- Algorithm designed so that states interfere to give one “number” with high probability

The Promise of a Quantum Computer

A Quantum Computer …

- Offers exponential improvement in \textit{speed} and \textit{memory} over existing computers
- Capable of \textit{reversible computation}
- e.g. Can factorize a 250-digit number in seconds while an ordinary computer will take 800 000 years!
1. Quantum Computing Roadmap Overview
2. Nuclear Magnetic Resonance Approaches
3. Ion Trap Approaches
4. Neutral Atom Approaches
5. Optical Approaches
6. Solid State Approaches
7. Superconducting Approaches
8. "Unique" Qubit Approaches
9. The Theory Component of the Quantum Information Processing and Quantum Computing Roadmap

Outline

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Circuits for Qubits

- Need to find circuits (dissipationless) which have two “good” energy levels
- Need to be able to “manipulate” qubits and couple them together

Harmonic Oscillator

\[ H = \frac{1}{2} m v^2 + \frac{1}{2} m \omega^2 x^2 \]

\[ v = \frac{dx}{dt} \]

\[ H = \frac{1}{2} m \left( \frac{dx}{dt} \right)^2 + \frac{1}{2} m \omega^2 x^2 \]

\[ p = m \frac{dx}{dt} \]

Quantum Mechanically

\[ \Delta x \Delta p \geq \frac{\hbar}{2} \]

\[ E = \hbar \omega \left( n + \frac{1}{2} \right) \]

LC Circuit

\[ H = \frac{1}{2} CV^2 + \frac{1}{2} LI^2 \]

\[ v = \frac{d\Phi}{dt} \quad \text{and} \quad I = \frac{\Phi}{L} \]

\[ H = \frac{1}{2} C \left( \frac{d\Phi}{dt} \right)^2 + \frac{1}{2} \frac{C}{L} \frac{1}{\omega^2} \Phi^2 \]

\[ p = C \frac{d\Phi}{dt} = CV \]

Quantum Mechanically

\[ L C \Delta I \Delta V \geq \hbar / 2 \]

\[ E = \hbar \omega \left( n + \frac{1}{2} \right) \]
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Quantization of Circuits

1. Find the energy of the circuit
2. Change the energy into the Hamiltonian of the circuit by identifying the canonical variables
3. Quantize the Hamiltonian
   ➢ Usually we can make it look like a familiar quantum system
Quantization of a Josephson Junction

Charging Energy

\[ U_c = \frac{1}{2} Q^2 C - \frac{1}{2} CV^2 \]

\[ = \frac{1}{2} \left( \frac{\Phi_0}{2\pi} \right)^2 C \left( \frac{\partial \phi}{\partial t} \right)^2 \]

\[ E_c = \frac{\Phi_0^2}{2C} \]

Josephson Energy

\[ U_j = \frac{\Phi_0 I_c}{2\pi} (1 - \cos \phi) \]

\[ E_j = \frac{\Phi_0 I_c}{2\pi} \]

Hamiltonian:

\[ H = \frac{1}{2} \left( \frac{\Phi_0}{2\pi} \right)^2 C \left( \frac{\partial \phi}{\partial t} \right)^2 + \frac{\Phi_0 I_c}{2\pi} (1 - \cos \phi) \]

Circuit behaves just like a physical pendulum.

For Al-Al₂O₃-Al junction with an area of 100x100 nm²
C = 1fF and \( I_c = 300 \text{ nA} \), which gives \( E_c = 10\mu\text{eV} \) and \( E_j = 600\mu\text{eV} \)

To see quantization, Temperature < 300 mK

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Proceed with $\psi$ as the coordinate:

\[ L = T - V \]  
Lagrangian

\[ p = \frac{\partial L}{\partial \dot{\psi}} \]  
canonical momentum

\[ H = p \dot{\psi} - L \]  
"Hamiltonian"

\[ I = \frac{1}{2} \left( \frac{\partial \psi}{\partial \phi} \right)^2 c \dot{\psi}^2 - E_J (1 - \cos \phi) \]

\[ p = \left( \frac{\partial \psi}{\partial \phi} \right) c \dot{\psi} \]

\[ H = \frac{c^2}{2} \dot{\psi}^2 + E_J (1 - \cos \phi) \]  
where $M = \frac{c^2}{2} \frac{\omega^2}{\beta^2}$

\[ \omega = c \nu, - \psi = \frac{2\pi}{\beta} p - c \dot{\psi} \]

It is convenient for physics, but not calculation, to

\[ L' = L - \left( \frac{\partial L}{\partial \dot{\psi}} \right) \]  
\[ \dot{\psi} \psi_d \]

\[ p' = p - \left( \frac{\partial L}{\partial \phi} \right) \]  
\[ \frac{\omega^2}{\beta^2} \]

\[ \dot{\psi} = \frac{\omega^2}{\beta^2} \]  
\[ p' \]

\[ H' = \frac{1}{2M} \left( p' + \frac{\omega^2}{\beta^2} \right)^2 + E_J (1 - \cos \phi) \]

\[ = \frac{1}{2} \left( \frac{\omega^2}{\beta^2} \right) \left( \gamma - \gamma \right)^2 + E_J (1 - \cos \phi) \]

Both $H$ & $H'$ describe the system.
Quantum Description

\[ A = \frac{\beta^2}{2m} + E_x (1 - \cos \beta) + \frac{2\pi \xi}{\hbar} \beta \cos \beta \]

Phase Picture \( \varphi = y \) \( \beta = \frac{\gamma}{\sqrt{2}} \sqrt{y} \) \( \Psi_0 = \Psi(y) \)

\[ \dot{\varphi} = -\frac{\beta^2}{2m} \frac{\partial^2}{\partial \varphi^2} + E_x (1 - \cos \varphi) + \frac{2\pi \xi}{\hbar} \frac{\partial}{\partial \varphi} \sin \frac{\gamma}{\sqrt{2}} \sqrt{y} \]

Charge Picture \( \beta = \frac{\gamma}{\sqrt{2}} \) \( y = -\frac{\gamma}{\sqrt{2}} \) \( \Psi_0 = \Psi(y) \)

\[ \dot{\varphi} = \frac{\beta^2}{2m} \frac{\partial^2}{\partial \varphi^2} + E_x (1 - \cos \varphi) + \frac{2\pi \xi}{\hbar} \frac{\partial}{\partial \varphi} \sin \frac{\gamma}{\sqrt{2}} \sqrt{y} \]

Note: \( \frac{e^{i\varphi}}{\Psi(y)} = \Psi(y + 1) \)

In either picture, we can write

\[ \frac{1}{\Psi} \frac{\partial}{\partial \varphi} \Psi = \frac{\partial^2}{\partial \varphi^2} \Psi \]

\( \varphi = \) states of definite \( \frac{1}{\Psi} \)

\( \frac{\delta}{\partial \varphi} \) "Change states"

\[ e^{i\beta \varphi} \]

\( \frac{\delta}{\partial \varphi} \) "Phase states"

Quantum Box

\[ \begin{align*}
\Psi_0 &= \frac{1}{\sqrt{2}} \Psi(y) \\
\frac{\gamma}{\sqrt{2}} &= \frac{1}{\sqrt{2}} \Psi(y) \\
\frac{\gamma}{\sqrt{2}} &= \frac{1}{\sqrt{2}} \Psi(y) \\
\Psi &= \frac{1}{\sqrt{2}} \Psi(y) \\
\end{align*} \]
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Dynamics Two-Level Quantum Systems

\[ H = \begin{pmatrix} -F & -V \\ -V^* & F \end{pmatrix} \]

\[ |\psi\rangle = \begin{pmatrix} \alpha \\ \beta \end{pmatrix} \]

Eigenenergies \( E = \sqrt{F^2 + V^2} \)

At \( F=0 \), let

\[ |\psi(t = 0)\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \]

\[ |\psi(t)\rangle = \frac{1}{2} \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{\frac{V_t}{\hbar} t} + \frac{1}{2} \begin{pmatrix} 1 \\ -1 \end{pmatrix} e^{-\frac{V_t}{\hbar} t} \]

\[ = \cos \left( \frac{V_t}{\hbar} t \right) \begin{pmatrix} 1 \\ 0 \end{pmatrix} + i \sin \left( \frac{V_t}{\hbar} t \right) \begin{pmatrix} 0 \\ 1 \end{pmatrix} \]

System oscillates between \( \begin{pmatrix} 1 \\ 0 \end{pmatrix} \) and \( \begin{pmatrix} 0 \\ 1 \end{pmatrix} \) with period \( T = \frac{\hbar}{2V} \)
Rabi Oscillations

Drive the system with \( V(t) = V_0 \, e^{i\omega t} \) at the resonant frequency \( \omega = E_+ - E_- \)

If \( |\psi(t = 0)\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \), then

\[
|\psi(t)\rangle = \cos \frac{V_0 t}{\hbar} \begin{pmatrix} 1 \\ 0 \end{pmatrix} + i \sin \frac{V_0 t}{\hbar} e^{i\omega t} \begin{pmatrix} 0 \\ 1 \end{pmatrix}
\]

Oscillations between states can be controlled by \( V_0 \) and the time of AC drive, with period

\[
T = \frac{\hbar}{2V_0}
\]

Charge-State Superconducting Qubit

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**Charge qubit**  
A Cooper-pair box $E_J / E_C \sim 0.3$

Coherence up to $\sim 5$ ns, presently limited by background charge noise (dephasing) and by readout process (relaxation)

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**Types of Superconducting Qubits**

- **Charge-state Qubits** (voltage-controlled)
  - Cooper pair boxes
- **Flux/Phase-state qubits** (flux-current control)
  - Persistent Current Qubits
  - RF SQUID Qubits
  - Phase Qubits (single junction)
- **Hybrid Charge-Phase Qubits**
Persistent current qubits require high-quality sub-micron junctions with low current density, and only MIT Lincoln has demonstrated this capability in Nb.

Energy Band Diagram of MIT-LL PC-Qubit: 1-20 GHz
Observation of Coherent Superposition of Macroscopic States
Jonathan Friedman, Vijay Patel, Wei Chen, Sergey Tolpygo and James Lukens

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Advantages of Superconductors for Quantum Computing

- Employs lithographic technology
- Scalable to large circuits
- Combined with on-chip, ultra-fast control electronics
  - Microwave Oscillators
  - Single Flux Quantum classical electronics
SQUID on-chip oscillator and qubit

On-chip oscillator couples to qubit: No spectroscopy yet due to high temperature

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Feasibility of Superconductive Control Electronics Fabrication

Image removed for copyright reasons.

On-chip Control for an RF-SQUID
M.J. Feldman, M.F. Bocko, Univ. of Rochester
www.ece.rochester.edu/~sde/

Images removed for copyright reasons.
Quantum Computation with Superconducting Quantum Devices


Dilution Refrigerator

Insert

Sample Holder

Massachusetts Institute of Technology

Installed and to begin dc data taking in February and ac data taking in April

DC Measurement on Nb Persistent Current Qubit

SQUID Detector

Magnetic Field (Ga)
Observation of Coherent Superposition of Macroscopic States

Jonathan Friedman, Vijay Patel, Wei Chen, Sergey Tolpygo and James Lukens

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CHARGE-FLUX QUBIT

Quantronics Group
CEA-Saclay
France

M. Devoret (now at Yale)
D. Esteve, C. Urbina
D. Vion, H. Pothier
P. Joyez, A. Cottet

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Coherence time measured by Ramsey fringes: 500ns
Qubit transition frequency: 16.5 GHz; coherence quality factor: 25,000

RAMSEY FRINGES

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