• i-v model for HMETs and HIGFETs (left from Lec. 12)  
  **DC model** - without velocity saturation  
  - with velocity saturation  
  Small signal linear equivalent circuit  
  High frequency limits - $\omega_T$, $\omega_{\text{max}}$

• Bipolar junction transistors (BJTs)  
  **Review of homojunction BJTs:**  
  i-v characteristics  
  small signal linear equiv. ckts.  
  high-f performance  

• Heterojunction bipolar transistors (HBTs)  
  **Historical perspective:** Shockley and Kroemer, motivation  
  A methodical look at heterojunction impacts:  
  - emitter issues  
  - base issues  
  - collector issues  

**Summary** - putting this together in a single device
To begin understanding and modeling BJTs we focus on finding
\[ i_E \left( v_{BE}, v_{BC} \right), \quad i_C \left( v_{BE}, v_{BC} \right) \]

Later we will also rearrange the expressions into a form that is more convenient for many applications:
\[ i_B \left( v_{BE}, v_{CE} \right), \quad i_C \left( v_{BE}, v_{CE} \right) \]
Ebers-Moll Model for BJT Characteristics

Divide the problem into two independent parts, forward and reverse:

\[ i_E(v_{BE}, v_{BC}) = i_{EF}(v_{BE}, 0) + i_{ER}(0, v_{BC}) \]

\[ i_C(v_{BE}, v_{BC}) = i_{CF}(v_{BE}, 0) + i_{CR}(0, v_{BC}) \]

We find for the forward portion:

\[ i_{EF}(v_{BE}, 0) = -Aqn_i^2 \left( \frac{D_e}{N_{AB}w_B^*} + \frac{D_h}{N_{DE}w_E^*} \right) \left( e^{qV_{BE}/kT} - 1 \right) \]

and:

\[ i_{CF}(v_{BE}, 0) = \left( 1 - \frac{w_B^*}{2L_e^2} \right) Aqn_i^2 \left( \frac{D_e}{N_{AB}w_B^*} \right) \left( e^{qV_{BE}/kT} - 1 \right) \]

These can be put in a more convenient form by defining emitter and base defects as:

\[ \delta_E \equiv \left( \frac{D_h}{D_e} \cdot \frac{w_B^*}{w_E^*} \cdot \frac{N_{AB}}{N_{DE}} \right), \quad \delta_B \equiv \frac{w_B^*}{2L_e^2} \]
Ebers-Moll Model for BJT Characteristics - cont.

We further define the emitter saturation current, \( I_{ES} \), and forward alpha, \( \alpha_F \):

\[
I_{ES} \equiv A q n_i^2 \left( \frac{D_e}{N_{AB} w_B^*} + \frac{D_h}{N_{DE} w_E^*} \right) \quad \text{and} \quad \alpha_F \equiv \frac{(1 - \delta_B)}{(1 + \delta_E)}
\]

Using these definitions we have:

\[
i_{EF}(v_{BE},0) = -I_{ES} \left( e^{qV_{BE}/kT} - 1 \right)
\]

\[
i_{CF}(v_{BE},0) = \alpha_F i_{EF}(v_{BE},0)
\]

Solving for the base current, we find:

\[
i_{BF}(v_{BE},0) = (1 - \alpha_F) i_{EF}(v_{BE},0)
\]

Finally, we note that it is particularly convenient to relate \( i_{BF} \) and \( i_{CF} \), by defining a forward beta, \( \beta_F \):

\[
i_{CF}(v_{BE},0) = \beta_F i_{BF}(v_{BE},0), \quad \text{with} \quad \beta_F \equiv \frac{\alpha_F}{(1 - \alpha_F)}
\]
Ebers-Moll Model for BJT Characteristics - cont.

Similarly, for the reverse portion we find:

\[ i_{ER}(0, v_{BC}) = -\alpha_R i_{CR}(0, v_{BC}) \]

\[ i_{CR}(0, v_{BC}) = -I_{CS} \left( e^{qV_{BC}/kT} - 1 \right) \]

Where we have made similar definitions:

\[ \delta_C \equiv \left( \frac{D_h}{D_e} \cdot \frac{w_B^*}{w_C^*} \cdot \frac{N_{AB}}{N_{DC}} \right) \]

\[ \delta_B \equiv \frac{w_B^*}{2L_e^2} \]

\[ I_{CS} \equiv Aqn_{i}^2 \left( \frac{D_e}{N_{AB}w_B^*} + \frac{D_h}{N_{DC}w_C^*} \right), \quad \alpha_R \equiv \frac{(1 - \delta_B)}{(1 + \delta_C)} \]

Combining the forward and reverse portions gives us the full characteristics:

\[ i_E(v_{BE}, v_{BC}) = -I_{ES} \left( e^{qV_{BE}/kT} - 1 \right) + \alpha_R I_{CS} \left( e^{qV_{BC}/kT} - 1 \right) \]

\[ i_C(v_{BE}, v_{BC}) = \alpha_F I_{ES} \left( e^{qV_{BE}/kT} - 1 \right) - I_{CS} \left( e^{qV_{BC}/kT} - 1 \right) \]
Ebers-Moll Model for BJT Characteristics - cont.

We primarily BJTs in their forward active region, i.e., $v_{BE} \gg kT/q$ and $v_{CE} \ll 0$, in which case we can usually neglect the reverse portion currents, and write

$$i_E(v_{BE}, v_{BC}) \approx -I_{ES}(e^{qV_{BE}/kT} - 1)$$

$$i_C(v_{BE}, v_{BC}) \approx \alpha_F I_{ES}(e^{qV_{BE}/kT} - 1)$$

$$i_B(v_{BE}, v_{BC}) \approx (1 - \alpha_F)I_{ES}(e^{qV_{BE}/kT} - 1)$$

Focusing on $i_B$ and $i_C$, we can in turn write:

$$i_B(v_{BE}, v_{BC}) \approx I_{BS}(e^{qV_{BE}/kT} - 1)$$

$$i_C(v_{BE}, v_{BC}) \approx \beta_F i_B(v_{BE}, v_{BC})$$

where we have use the forward beta, $\beta_F$ we defined earlier.
One objective of BJT design is to get a large forward beta, $\beta_F$. To understand how we do this we write beta in terms of the defects, and then in terms of the device parameters:

$$\beta_F = \frac{\alpha_F}{1 - \alpha_F} = \frac{1 - \delta_B}{\delta_E + \delta_B} \approx \frac{1}{\delta_E} = \left( \frac{D_e}{D_h} \cdot \frac{w^*_E}{w^*_B} \cdot \frac{N_{DE}}{N_{AB}} \right),$$

In getting this we have assumed that $\delta_B \ll \delta_E$, which is generally true if the base width is small:

$$L_{eB} \gg w^*_B \rightarrow \delta_B \text{ is negligible}$$

From our $\beta_F$ result we see immediately the design rules for BJTs:

$$N_{DE} \gg N_{AB}, w^*_B < w^*_E, \text{ npn preferred over pnp}$$
Ebers-Moll Forward, $v_{BE} > 0$, $v_{BC} = 0$

Excess Carriers:

\[
(n_i^2/N_{DE})(e^{q_{VBE}/kT} - 1)
\]

Currents:

\[
i_e = i_e E (1 + \delta_E)
\]
\[
i_h = \delta_E i_e E
\]
\[
-i_C = i_e E (1 - \delta_B)
\]
\[
-i_B = i_e E (\delta_E + \delta_B)
\]
Well designed structure: $N_{DE} > N_{AB}$, $w_E << L_{hE}$, $w_B << L_{eB}$

**Excess Carriers:**

$$\frac{n_i^2}{N_{DE}}(e^{qV_{BE}/kT} - 1)$$

0 (ohmic)

$-w_E$ 0 $w_B$ $w_B + w_C$

**Currents:**

$$i_E = i_E(1 + \delta_E)$$

$$i_{hE} = \delta_E i_E$$

$$i_C = i_E(1 - \delta_B) \approx i_E$$

$$i_B = -i_E(\delta_E + \delta_B) \approx -i_E\delta_E$$

C. G. Fonstad, 4/03

Lecture 13 - Slide 9
BJT Characteristics (nnp)

**Forward Active Region**:
- $v_{BE} > 0.6 \text{ V}$
- $v_{CE} > 0.2 \text{ V}$
- (i.e. $v_{BC} < 0.4 \text{ V}$)
- $i_R$ is negligible

**Cutoff**:
- $v_{BE} < 0.6 \text{ V}$

**Saturation**:
- $v_{CE} < 0.2 \text{ V}$

**BJT Models**

$\alpha_F i_F$ is the forward active region current, and $\beta_F i_b$ is the base current.
Using Heterojunctions to improve BJTs:

**Historical Note:**

The first ideas for using heterojunctions in BJTs was directed at increasing beta by making the emitter defect smaller. This possibility was implied by Shockley in his original transistor patent (c. 1950) and proposed explicitly by Herb Kroemer in a 1958 paper. He reproposed the idea in another paper about 1980, when it was technologically possible to explore this idea, and that is when heterojunction bipolar transistors, HBTs, took off.

To investigate the full potential of heterojunctions in BJTs we will look at the issue somewhat more broadly.

There are three main pieces of a bipolar transistor, and we will look at issues involved with each one:

- Emitter issues
- Base issues
- Collector issues
Emitter issues:

In a homojunction transistor the lateral conductivity of the base is limited by the doping level and base width. It could be increased if the base doping level could be increased, but this impacts the emitter defect negatively.

With a wide bandgap emitter layer, the coupling another factor is introduced in the emitter defect which reduces the need to dope the base heavily.

In a homojunction:

$$\delta_E \equiv \left( \frac{D_h}{D_e} \cdot \frac{w_B^*}{w_E^*} \cdot \frac{N_{AB}}{N_{DE}} \right)$$

In a heterojunction:

$$\delta_E \equiv \left( \frac{D_h}{D_e} \cdot \frac{w_B^*}{w_E^*} \cdot \frac{N_{AB}}{N_{DE}} \right) e^{-(HB-EB)/kT}$$

New factor:  
- HB = hole barrier 
- EB = electron barrier

Note: We saw this when we talked about heterojunctions in Lecture 4.
Emitter issues: Hole barrier vs. electron barrier

The size of the barrier depends on whether or not there is an effective spike in one of the bands (usually in the conduction band)

NOTE: In an N-p+ junction, all of the band bending will be on the N-side.

\[ \Delta E_B \approx q \Delta \phi \]

If the spike is a barrier: \( HB - EB \approx \Delta E_C \)
Emitter issues: Hole barrier vs. electron barrier

If the spike can be made thin enough to tunnel, or (better) is eliminated by grading the composition at the heterojunction, then HB - EB can be increased significantly:

\[ \approx q \Delta \phi - \Delta E_c \]

\[ \approx q \Delta \phi + \Delta E_v \]

If the spike is not a barrier: \( HB - EB \approx \Delta E_c + \Delta E_v = \Delta E_g \)

NOTE: Grading removes the spike without requiring heavy N-doping.
Emitter issues: Dopant diffusion from base

In a heterojunction transistor the base is usually very heavily doped, while the emitter is more lightly doped. In such a case it is easy for dopant to diffuse into the emitter and dope it P-type. This is very bad:

A solution is to introduce a thin, undoped wide bandgap spacer.
Emitter issues: Using the barrier to advantage
Ballistic injection

The concept of ballistic injection is that the carriers going over the spike will enter the base with a high initial velocity: $E_i$

This can reduce the base transit time.
A problem is that if the carriers are given too much initial energy they will scatter into a higher energy band and slow down.
Base issues:  Reducing the base resistance
Reducing the base transit time

**Base resistance:**
dope very heavily

**Base transit time:**
make base very thin
grade the base to build in
a field for the electrons

Note: If the base is very thin it may be difficult to grade the composition very much.

As with ballistic injection, a problem is that if the carriers get too much energy from the field they can scatter into a higher energy band and slow down.
Collector issues:

**Base width modulation:** not an issue because of heavy base doping

**Avalanche breakdown:** an issue with InGaAs bases and collectors

  **Solution:** wide band gap collector

**Turn-on/Saturation off-set:** can be significant with a homojunction collector

  **Solution:** wide band gap collector
Collector issues, cont: a caution about wide bandgap collectors

A wide bandgap collector can be a problem if there is a conduction band spike at the interface. The collector won't collect at low reverse biases!!

The solution is to grade the base-collector interface.
HBT Processing: a typical cross-section

We'll look extensively at processes and circuits our next in lecture (#14).