Dissipation and
The Thermal Energy Domain Part I

Joel Voldman*
Massachusetts Institute of Technology
*(with thanks to SDS)
Outline

> Thermal energy domain: why we love it and why we hate it

> Dissipative processes
  • Example: Charging a capacitor through a resistor

> The Thermal Energy Domain
  • Governing equations

> Equivalent-circuit elements & the electrothermal transducer

> Modeling the bolometer
Thermal MEMS

> Everything is affected by temperature

> Therefore, anything can be detected or measured or actuated via the thermal domain

> Sometimes this is good…
MEMS Imagers

> A bolometer heats up due to incoming radiation

> This results in a temperature change that changes the resistance across the pixel

Poor residual stress control

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Better design…

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MEMS Imagers

> This application illustrates key features of thermal MEMS

- Excellent thermal isolation creates excellent sensitivity
  » Response is proportional to thermal resistance
- Low thermal mass creates fast response time
  » Response time is proportional to thermal capacitance
- Easy integration with sense electronics
Thermal flow sensing

> A time-of-flight flowrate sensor
> One resistor creates a heat pulse
> A downstream resistor acts as a temperature sensor
> Time for heat pulse to drift downstream is inversely related to flowrate
> This example illustrates the important benefit of MEMS materials
  • Large range in thermal conductivities
  • From vacuum (~0) to metal (~100’s W/m-K)


MEMS flow sensing: commercial

> OMRON flow sensor uses measures temperature distribution around a heat source

> Convection alters temperature profile in a predictable fashion
The Thermal Domain

> Sometimes this is bad…

> Reason #1

• Everything is a temperature sensor
• Evaluation of MEMS devices over temperature is often critical to success

> Reason #2

• As we will see, we can never transfer energy between domains perfectly
• The “extra” energy goes into the thermal domain (e.g., heat)
• We can never totally recover that heat energy

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Charging a capacitor

> We will transfer energy from a power supply to a capacitor

> Ideally, all energy delivered from supply goes to capacitor

> In actuality, there is ALWAYS dissipation
  • And this is true for ALL domains

> Thus, we will lose some energy
Example: Charging a Capacitor

> Use step input

> Voltage source must supply twice the amount of energy as goes into the capacitor

> One half the energy is dissipated in the resistor, independent of the value of $R$!

Energy stored in capacitor: $W_C = \frac{1}{2} CV_S^2$

Energy delivered by power supply:

$$P_S(t) = IV_s = \frac{V_S^2}{R} e^{-t/RC}$$

$$W_S = \int_0^\infty P_S dt = CV_S^2$$
Power Considerations: Joule Heating

> The extra energy is lost to Joule heating in the resistor

> **Globally**, the power entering a resistor is given by the $IV$ product.

> **Locally**, there is power dissipation given by the product of the charge flux and the electric field.

\[ P_R = IV_R = I^2R = \frac{V^2}{R} \]

For normal "positive" resistors \( P_R \geq 0 \)

This means \[ [ \text{J}/\text{m}^3 \]

\[ P_R^0 = J_eE = \sigma_eE^2 \]

For normal positive conductivity \( P_R^0 \geq 0 \)

and, one is the integral of the other

\[ P_R = \iiint_{Volume} P_R^0 d^3r \]
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Thermal Energy Domain

> It is easier to put ENERGY INTO than get WORK OUT of thermal domain

> All domains are linked to thermal domain via dissipation

> Thermal domain is linked to all domains because temperature affects constitutive properties

Thermal Energy Domain

> Heat engines convert heat into mechanical work, but not perfectly efficiently, just like the charging of a capacitor cannot be done perfectly efficiently

> This is a statement of irreversibility: the 2nd Law of Thermodynamics

> Zyvex heatuator
  • One skinny leg and one fat leg
  • Run a current and skinny leg will heat up
  • Structure will bend in response

Courtesy of Zyvex Corporation. Used with permission.
Governing Equations

> Some introductory notation

> Be careful with units and normalizations

\[ Q \quad \text{Thermal energy [J]} \]

\[ \tilde{Q} \quad \text{Thermal energy/volume [J/m}\(^3\)] \]

\[ \dot{Q} = I_Q \quad \text{Heat flow [W]} \]

\[ J_Q \quad \text{Heat flux [W/m}^2\text{]} \]

\[ C_V = \frac{\partial Q}{\partial T}\bigg|_{\text{Volume}} \]

Heat capacity at constant volume (J/K)

\[ C_P = \frac{\partial Q}{\partial T}\bigg|_{\text{Pressure}} \]

Heat capacity at constant pressure (J/K)

Are the same for incompressible materials

\[ C_V = C_P = C \]

Heat capacity/unit volume (J/K-m\(^3\))

\[ \tilde{C} = \frac{C}{V} \]

Heat capacity/unit mass (J/K-kg) AKA specific heat

\[ \tilde{C}_m = \frac{\tilde{C}}{\rho_m} \]
Governing Equations

> Like many domains, conservation of energy leads to a continuity equation for thermal energy

\[
\frac{d}{dt} \left( \frac{\text{stuff in volume}}{\text{volume}} \right) = \left( \frac{\text{net stuff entering volume}}{\text{volume}} \right) + \left( \frac{\text{generation of stuff in volume}}{\text{volume}} \right)
\]

\[
\frac{d}{dt} \int bdV = -\int F \cdot n dS + \int gdV
\]

Get point relation

Divergence theorem and bring derivative inside

\[
\int b \frac{\partial}{\partial t} dV = -\int \nabla \cdot F dV + \int gdV
\]

\[
\frac{\partial b}{\partial t} = -\nabla \cdot F + g
\]

For heat transfer

\[
\frac{dQ}{dt} + \nabla \cdot J_Q = \tilde{P}_{\text{sources}}
\]
Types of Heat Flow

> Flow proportional to a temperature gradient
  • Heat conduction
  \[ J_Q = -\kappa \nabla T \]

> Convective heat transfer
  • A subject coupling heat transfer to fluid mechanics
  • Often not important for MEMS, but sometimes is…
  • Talk more about this later
  \[ J_Q = h_c (T_2 - T_1) \]

> Radiative heat transfer
  • Between two bodies (at $T_1$ and $T_2$)
  • Stefan-Boltzmann Law
  • Can NEVER turn off
  \[ J_Q = \sigma_{SB} F_{12} (T_2^4 - T_1^4) \]
The Heat-Flow Equation

> If we assume linear heat conduction, we are led to the heat-flow equation

\[
J_Q = -\kappa \nabla T
\]

\[
\nabla \cdot J_Q = -\nabla \cdot (\kappa \nabla T)
\]

\[
\frac{\partial \tilde{Q}}{\partial t} = \nabla \cdot (\kappa \nabla T) + \tilde{P}_{\text{sources}}
\]

For homogeneous materials, with \( \frac{d\tilde{Q}^0}{dT} = \tilde{\alpha} \),

\[
\frac{\partial T}{\partial t} = \frac{\kappa}{C} \nabla^2 T + \frac{1}{C} \tilde{P}_{\text{sources}}
\]
Thermodynamic Realities

> The First law of thermodynamics implies that entropy is a generalized displacement, and temperature is a generalized effort; their product is energy

> The Second Law states that entropy production is $\geq 0$ for any process
  * In practice it always increases

> Entropy is not a conserved quantity....

> Thus, it does not make for a good generalized variable

> Therefore, we use a new convention, the thermal modeling convention, with temperature as effort and heat flow (power) as the flow. Note that the product of effort and flow is no longer power!!!
  * But heat energy (thermal displacement) is conserved
  * Just like charge (electrical displacement) is conserved
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Thermal sources

> Heat current source $I_Q$ is represented with a flow source

> Temperature difference source $\Delta T$ is represented with an effort source
Thermal equivalent-circuit elements

> Use direct analogy again

Thermal conductivity

\[ J_Q = -\kappa \nabla T \]

Relation between effort and flow

\[ J_E = \sigma_e E = -\sigma_e \nabla V \]

Laplace’s Eqn.

\[ \nabla^2 T = 0 \]

\[ \nabla^2 V = 0 \]

Continuity of effort

\[ T_2 = T_1 \]

\[ V_2 = V_1 \]

Continuity of flow

\[ (J_Q \cdot n)_1 = (J_Q \cdot n)_2 \]

\[ (J_E \cdot n)_1 = (J_E \cdot n)_2 \]

No charge storage

No heat storage

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Thermal equivalent-circuit elements

> Therefore we can derive a thermal resistance

\[ I_Q \quad R_T \quad \Delta T \]

\[ R_T = \frac{1}{\kappa A} \quad [K/W] \quad \text{For bar of uniform cross-section} \]

> Plus, heat conduction and current flow obey the same differential equation

> Thus, we can use exactly the same solutions for thermal resistors as for electrical resistors

• Just change \( \sigma \) to \( \kappa \)
Thermal equivalent-circuit elements

> What about other types of heat flow?

> Convection

> Use linear resistor

\[ J_Q = h_c (T_2 - T_1) \]

\[ I_Q = h_c A (T_2 - T_1) = h_c A \Delta T \]

\[ R_{T,\text{conv}} = \frac{1}{h_c A} \]
Thermal equivalent-circuit elements

> Radiation

- Nonlinear with temperature

Large-signal model

\[ J_Q = \sigma_{SB} F_{12} \left( T_2^4 - T_1^4 \right) \]

\[ I_Q = \sigma_{SB} F_{12} A \left( T_2^4 - T_1^4 \right) \]
> Radiation

• Many ways to linearize
• We show two approaches

\[ I_Q = \sigma_{SB} F_{12} A \left( T_2^4 - T_1^4 \right) \]

\[ T_2 \gg T_1 \]

\[ I_Q \approx \sigma_{SB} F_{12} A \left( T_1 + \delta T \right)^4 - T_1^4 \]

\[ \delta I_Q = \left( 4 \sigma_{SB} F_{12} A T_1^3 \right) \delta T \]

\[ T_2 \approx T_1 \]
 Thermal equivalent-circuit elements

> What about energy storage?

> Just like electrical capacitors store charge \( (Q) \),

> We can store thermal energy \( (Q) \)

> No thermal inductor 😞

\[
C_E = \frac{\partial Q}{\partial V}
\]

\[
C_T = \frac{\partial Q}{\partial T}
\]

\[
C_T = \tilde{C}_m \rho_m V
\]

[Diagram of thermal equivalent circuit]
Electro-thermal transducer

> In electromechanical energy transduction, we introduced the two-port capacitor
  • Energy-storing element coupling the two domains
  • Capacitor because it stores potential energy

> What will we use to couple electrical energy into thermal energy?
  • In the electrical domain, this is due to Joule dissipation, a loss mechanism
    » Therefore it looks like an electrical resistor
  • In the thermal domain it looks like a heat source
    » Therefore it looks like a thermal current source
Electro-thermal transducer

> Our transducer is a resistor and a dependent current source
  • The thermal current source depends on $R$ and $I$ in the electrical domain

> We reverse convention on direction of port variables in thermal domain
  • OK, because $I_Q \cdot \Delta T$ does not track power
  • Reflects fact that heat current will always be positive out of transducer

> This is not energy-conserving
  • Dissipation is intrinsic to transducer

> This is not reciprocal
  • Heat current does not cause a voltage

> Thermal domain can couple back to the electrical domain
  • See next time...
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Different heat transfer components

> How does one know which heat transfer processes are important for thermal modeling?

Heat input is a current
Heat loss is a resistor
Heat storage in mass is a capacitor

\[ \Delta T = I_Q \left( R_T \parallel \frac{1}{C_T s} \right) = \frac{R_T I_Q}{1 + R_T C_T s} \]

\[ \Delta T_{ss} = R_T I_Q \]
\[ \tau = R_T C_T \]
Implications

> Increasing $R_T$ increases response
  - This means better thermal isolation
  - There is always some limiting value determined by radiation

> Given a fixed $R_T$, decreasing $C_T$ improves response time
  - This means reducing the mass or volume of the system

\[
\Delta T_{ss} = R_T I_Q \\
\tau = R_T C_T
\]
Thermal resistance

> What goes into $R_T$?

> $R_T$ is the parallel combination of all loss terms

• Conduction through the air and legs
• Convection
• Radiation

> We can determine when different terms dominate
Conduction resistance

> Conduction

• Si is too thermally conductive
• SiO₂ is compressively stressed
• Try SiN

<table>
<thead>
<tr>
<th>Material</th>
<th>κ (W/m-K)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Silicon</td>
<td>148</td>
</tr>
<tr>
<td>Silicon Nitride</td>
<td>20</td>
</tr>
<tr>
<td>Thermal Oxide</td>
<td>1.5</td>
</tr>
<tr>
<td>Air (1 atm)</td>
<td>0.03</td>
</tr>
<tr>
<td>Air (1 mtorr)</td>
<td>10⁻⁵</td>
</tr>
</tbody>
</table>

\[ R_{T,\text{legs}} = \frac{1}{2} \frac{L}{\kappa A} \]
\[ = \frac{(50 \, \mu m)}{2(20 \, \text{W/m-K})(0.5 \, \mu m)(5 \, \mu m)} \]
\[ R_{T,\text{legs}} = 5 \cdot 10^5 \, \text{K/W} \]

\[ R_{T,\text{air}} = \frac{1}{\kappa A} \]
\[ = \frac{(2.5 \, \mu m)}{(10^{-5} \, \text{W/m-K})(50 \, \mu m)(50 \, \mu m)} \]
\[ R_{T,\text{air}} = 10^8 \, \text{K/W} \]

Conduction through legs dominates
Other resistances

> **Convection**
  
  • At low pressure, there will be no air movement
  • Convection will not exist

> **Radiation**
  
  • There is transfer between plate and body
  • Use case where bodies are close in temp
  • Radiation negligible

> **Very fast time constant**
  
  • ~ms is typical for thermal MEMS

\[ R_{T,rad} = \frac{1}{4\sigma_{SB} F_{12} A T_1^3} \]

\[ R_{T,rad} = \frac{1}{4(5.67 \cdot 10^{-8} \text{ W/m}^2\text{K}^4)(0.5)(50 \mu m)(50 \mu m)(300 \text{ K})^3} \]

\[ R_{T,legs} = 1.3 \cdot 10^8 \text{ K/W} \]

**Leg conduction dominates loss**

\[ C_T = \tilde{C}_m \rho_m V \]

\[ = (700 \text{ J/kg-K})(3000 \text{ kg/m}^3)(50 \mu m)(50 \mu m)(0.5 \mu m) \]

\[ = 3 \cdot 10^{-9} \text{ J/K} \]

\[ R_T C_T = 1.5 \text{ ms} \]
Improving the design

How can we make responsivity higher?

Change materials

Decrease thickness or width of legs, or increase length
  • This reduces mechanical rigidity
Does convection ever matter?

> Usually not

> But it can come into play in microfluidics

> The key is whether energy is transported faster by fluid flow or heat conduction

> We’ll analyze this a bit better after we do fluids
The Next Step

> When dealing with conservative systems, we found general modeling methods based on energy conservation

> With dissipative systems, we must always be coupled to the thermal energy domain, and must address time-dependence

> This is the topic for the next Lecture