6.866

Quiz #1 Oct 19 Quiz #2 Nov 30
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6) Estimating motion

1st example, a vector \((x,y)\), a variable

\[ E = \text{brightness} \]

\[ x = \text{position along the image} \]

\[ \text{shift + image noise} \]

\[ u = \text{velocity} = \frac{\Delta x}{\Delta t} \]

\[ \Delta x = u \cdot \Delta t \]

\[ \frac{dE}{dx} = \frac{-\Delta E}{u \cdot \Delta t} \]

\[ u \cdot (E_x + E_t) = 0. \]

or: \[ u \frac{dE}{dx} + \frac{\Delta E}{\Delta t} = 0 \]

\[ E_x = \frac{E_{x_1} - E_1}{\Delta x} \quad \text{and} \quad E_t = \frac{E_{x_1} - E_1}{\Delta t} \quad \text{Thus} \]

\[ u = \frac{-E_t}{E_x} \]

but image noisy so poor method

and what if \( E_t = 0 \)? (blank area)

and what if \( E_t \approx 0 \)? not good either

Belius method:

\[ u \approx \frac{-1}{x_2 - x_1} \int_{x_1}^{x_2} E_t \, dx + \text{filtering} \ E_t \approx 0 \]
\[ u = -\frac{1}{x_2 - x_1} \int_{x_1}^{x_2} w(x) \frac{E_x}{E_t} \, dx \quad \text{where} \quad W = \frac{1}{x_2 - x_1} \int_{x_1}^{x_2} w(x) \, dx \]

so \[ u = -\int_{x_1}^{x_2} w(x) \frac{E_x}{E_t} \, dx \div \int_{x_1}^{x_2} w(x) \, dx \]

let's take \( w(x) = E_t \) \[ u = -\int_{x_1}^{x_2} E_x \, dx \div \int_{x_1}^{x_2} E_t \, dx \quad \text{not good!} \]

but what about \( w(x) = |E_t| \)

\[ u = -\int_{x_1}^{x_2} E_x \, dx \div \int_{x_1}^{x_2} |E_t| \, dx \quad \text{not optimal... discontinuities} \]

so \( w(x) = E_t^2 \)

\[ u = -\int_{x_1}^{x_2} E_x E_t \, dx \div \int_{x_1}^{x_2} E_t^2 \, dx \quad \text{\( \Theta \) takes info from all the image} \]

Another way:

\[ u E_x + E_t = 0 \quad \text{constraint equation} \]

\[ \int_{x_1}^{x_2} (uE_x + E_t)^2 \, dx \quad \text{\( \rightarrow 0 \) perfect case} \]

\[ \int_{x_1}^{x_2} (uE_x + E_t)^2 \, dx \quad \text{\( \rightarrow 0 \) perfect case} \]
\( u = \min \text{ value for which } E \)
\[ \frac{d}{du} (\text{something}) = 0 \implies \int_{x_1}^{x_2} 2E_x (uE_x + E_t) E_x \, dx = 0 \]
\( u \int_{x_1}^{x_2} E_x^2 \, dx + \int_{x_1}^{x_2} E_x E_t \, dx = 0 \)
\[ u = -\int_{x_1}^{x_2} E_x E_t \, dx \Bigg/ \int_{x_1}^{x_2} E_x^2 \, dx \]

Least Square Solution
"optical flow"

\[ \text{If motion is constant, then average over time also} \rightarrow \text{more accurate} \]

2) Image projection
pinhole model
\[ (x, y, z)^T \rightarrow (x', y', f)^T \]

\[ f = \text{dist. bet. hole} \& \, \text{CCD principal point} \]

2D

3D

\[ \frac{x}{f} = \frac{X}{Z} \quad \frac{y}{f} = \frac{Y}{Z} \]

\[ f = \text{zoom factor} + \text{scale ambiguity} \]

perspective projection equation
(1) Brightness

Parameters:
(i) material of surface
(ii) illumination: power & distribution
(iii) viewer direction

Model of a particular type of surface: "Lambertian" (ideal)

(i) reflects all incident light
(ii) appears equally bright from all viewpoints (mat)

\[
\frac{P}{A} = \frac{P'}{A'} = P \cos \theta
\]
1. A line in 3D becomes a line in 2D camera.
2. Two parallel lines in 3D are generally not parallel in 2D.
3. The common point is the vanishing point.

Equation of a line:

\[
\begin{pmatrix}
  x \\
  y \\
  z
\end{pmatrix} = \begin{pmatrix}
  x_i \\
  y_i \\
  z_i
\end{pmatrix} + \begin{pmatrix}
  \frac{R_2}{R_e} \\
  \frac{R_1}{R_e}
\end{pmatrix}
\]

\[
\| (a, b, c) \| = 1
\]

\[
R_{(a, b, c)}
\]

\[
\begin{pmatrix}
  \frac{x}{R} \\
  \frac{y}{R}
\end{pmatrix} = \begin{pmatrix}
  \frac{x_i}{R_i} + \frac{R_2}{R_e} \\
  \frac{y_i}{R_i} + \frac{R_1}{R_e}
\end{pmatrix}
\]

\[
\text{If } R \rightarrow \infty, \quad \frac{x}{R} \rightarrow \frac{x_i}{R_i}, \quad \frac{y}{R} \rightarrow \frac{y_i}{R_i}
\]

**Vanishing point**