Quiz #1  Reminder in continuous images

If edge is curved?

Generalize:

\[
P = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \sqrt{E_x^2 + E_y^2} \, dx \, dy
\]

\[
\frac{\partial E}{\partial x} = \frac{\partial E}{\partial \epsilon} \frac{\partial \epsilon}{\partial x} + \frac{\partial E}{\partial \theta} \frac{\partial \theta}{\partial x} = \epsilon'(x) \cos \theta
\]

\[
r = \sqrt{x^2 + y^2} \Rightarrow \frac{\partial r}{\partial x} = \frac{x}{\sqrt{x^2 + y^2}} = \frac{\epsilon'(x) \cos \theta}{r}
\]

\[
\int_{-\pi}^{\pi} \int_{R_{\epsilon}}^{R_{\epsilon+\epsilon}} (\epsilon'(x)) \, rd\theta dx
\]
\[
\text{as } \varepsilon \to 0, \quad 2 \pi \int_0^\pi R(\varepsilon) R_0 \, d\varepsilon = \int_0^\pi R_0 \left( R_0(\varepsilon + \varepsilon) - R_0(\varepsilon - \varepsilon) \right) \, d\varepsilon
\]

Circular arc tangent continuous approximation

Area: \( A = \int \int E(x, y) \, dx \, dy \)

Property

\[
A(x) + A(y) = A(x \cup y) - A(x \cap y)
\]

Area satisfies ASP.

\[
A(x \cup y) - A(x) = A(y) - A(x \cap y)
\]

\[
A(x \cup \Delta x) - A(x) = A(\Delta x) - A(x \cap \Delta x)
\]

\[
A(x \cap \Delta x) - A(x) = A(\Delta x) - A(x \cap \Delta x)
\]

\[
\Delta A^m = A(\Delta x^m) - A(x \cap \Delta x^m)
\]

ex: \( \Delta x^m \) sample pixel
Stream direchon and slice the image into thin slices:

\[ \text{Area: } x^m \Delta x^m = 0 \]
\[ A = \sum \Delta A^m = \sum A(\Delta x^m) \]

\[ \text{Perimeter: } P = \sum \Delta P^m - \sum P^m \Delta P^m \]

Euler number
\[ E = m(\text{object}) - m(\text{holes}) \]

\[ E = 1 - 2 = -1 \]
\[ E = 3 \]

Algorithm

1. \[ E_1 = 1 - 0 = 0 \] (ASP)
2. \[ E_2 = E_1 + 2 - 1 = 2 \]
3. \[ E_3 = E_2 + 2 - 2 = E_2 \] etc...
4. \[ E_4 = 0 \] ??? How to improve?

\[ E = m(\text{object}) - m(\text{holes}) \]

\[ E = 1 \]
Back to discrete world

\[ m(0,0) - m(0,1) \]

\[ \text{stream} \]

\[ N_0 \]

\[ N_1 \]

\[ N_2 \]

\[ N_3 \]

\[ G - 	ext{connectiveness} \]

\[ E = \frac{1}{6} (N_1 - N_2) \]

\[ P = \frac{N_1}{6} (N_0 + N_2) \]

\[ A = \frac{N_1}{6} (N_0 + 2N_2 + 3N_3) \]

\[ E = 1 \quad P = 1 \quad A = 1 \]

\[ N_0 = 0 \quad N_1 = 0 \quad N_2 = 0 \quad N_3 = 0 \]

\[ N_0 = 0 \quad N_1 = 9 \quad N_2 = 3 \quad N_3 = 1 \]

\[ E = 1 \quad P = 12 \quad A = 3 \]
Iterative Mohr's Computation

Euler Differential

How does \( E \) change as one changes one pixel? \( E^* = \frac{\Delta E}{\Delta T} \)

Safe change | Unsafe change

Hexagonal grid

Very hard on square grid!

\( 2^6 = 64 \) patterns + rotational symmetry + mirror images

\( E^* = 0 \)

\( E^* = -2 \)

\( E^* = -1 \)

\( E^* = +1 \)

\( E^* = +1 \)
Problem

surrounding operator

\[ \text{how many possible functions? } 2^4 = 16 \]

\[ a \rightarrow \text{ detect local feature} \]
\[ a \leq b \text{ monotonic } \quad \text{"thinning"} \]
\[ a + b \geq b \text{ monotonic } \quad \text{"swelling"} \]
\[ a \circ b \text{ not monotonic } \rightarrow \text{Conway's game of life} \]

Wolfram: A new kind of science