Explicit State Model Checking

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Explicit State Model checking

The basic Strategy

Temporal Logic Formula

Buchi Automata

Product Automata

Model checker

OK

Kripke structure

System Description

Counterexample trace
A Buchi Automaton is a 5-tuple $\langle \Sigma, S, I, \delta, F \rangle$
- $\Sigma$ is an alphabet
- $S$ is a finite set of states
- $I \subseteq S$ is a set of initial states
- $\delta \subseteq S \times \Sigma \times S$ is a transition relation
- $F \subseteq S$ is a set of accepting states

Non-deterministic Buchi Automata are not equivalent to deterministic ones
Buchi Automaton from Kripke Structure

Given a Kripke structure:
- $M = (S, S_0, R, L)$

Construct a Buchi Automaton
- $(\Sigma, S \cup \{Init\}, \{Init\}, T, S \cup \{Init\})$
- $T$ is defined s.t.
  - $T(s, \sigma, s')$ iff $R(s, s')$ and $\sigma \in L(s')$
  - $T(Init, \sigma, s)$ iff $s \in S_0$ and $\sigma \in L(s)$
Buchi Automaton from Kripke Structure

- \((\Sigma, S \cup \{\text{Init}\}, \{\text{Init}\}, T, S \cup \{\text{Init}\})\)
- \(T\) is defined s.t.
  - \(T(s, \sigma, s')\) iff \(R(s, s')\) and \(\sigma \in L(s')\)
  - \(T(\text{Init}, \sigma, s)\) iff \(s \in S_0\) and \(\sigma \in L(s)\)
**Buchi Automaton from Kripke Structure**

**Given a Kripke structure:**
- \( M = (S, S_0, R, L) \)

**Construct a Buchi Automaton**
- \( (\Sigma, S \cup \{\text{Init}\}, \{\text{Init}\}, T, S \cup \{\text{Init}\} ) \)

- \( T \) is defined s.t.
  - \( T(s, \sigma, s') \) iff \( R(s, s') \) and \( \sigma \in L(s') \)
  - \( T(\text{Init}, \sigma, s) \) iff \( s \in S_0 \) and \( \sigma \in L(s) \)

**What about missing transitions?**
- Need to add a dummy “error state”
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Counterexample trace

Kripke structure

System Description
Negated Properties

Given a good property $P$, you can define a bad property $P'$
- If the system has a trace that satisfies $P'$, then it is buggy.

Example
- Good property: $G(\text{req} \Rightarrow \text{F ack})$
- Bad property: $F(\text{req} \& \neg (\text{G !ack}))$

We are going to ask whether $M$ satisfies $P'$
- If it does, then we found a bug

Why are we doing the negation?
Computing the Product Automata

Given Buchi automata $A$ and $B'$

- $A = (\Sigma, S_A, T_A, \{\text{Init}_A\}, S_A)$
- $B' = (\Sigma, S_B, T_B, \{\text{Init}_B\}, F')$
- $A \times B' = (\Sigma, S_A \times S_B, T, \{(\text{Init}_A, \text{Init}_B)\}, F)$

Where

- $T((s_1, s_2), \sigma, (s_1', s_2'))$ iff $T_A(s_1, \sigma, s_1')$ and $T_B(s_2, \sigma, s_2')$
- $(s_1, s_2) \in F$ iff $s_2 \in F'$
Check if a state is visited infinitely often

Check for a cycle with an accepting state

Cycle must be reachable from the initial state

Simple algorithm
- Do DFS to find an accepting state
- Do a DFS from that accepting state to see if it can reach itself
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The basic Strategy

Temporal Logic Formula → Buchi Automata → Product Automata → Model checker

Kripke structure → DFS on state machine!

Counterexample trace → OK
Example

$G \text{ rec} \rightarrow F \text{ ack}$
Optimizations: Partial Order Reduction

**Example**

```
while(*){
    pc=0  if(p=0){
        p:=1;
    }  
    pc=1  
    pc=2  if(p=1){
        if(g=free){
            g:=id;
            p:=2;
        }
    }
    pc=3  
    pc=4  if(g=free){
        g:=id;
        p:=2;
    }
    pc=5
    pc=6  if(p=2){
        p:=3; g:=free
    }
    pc=7
    pc=8  if(p=3){
        p:=0;
    }  
    pc=9
}
```

```
while(*){
    if(p=0){
        p:=1;
    }
    if(p=1){
        if(g=free){
            g:=id;
            p:=2;
        }
    }
    if(p=2){
        p:=3; g:=free
    }
    if(p=3){
        p:=0;
    }
}
```
Optimizations: Partial Order Reduction

Example

```
while(*){
    pc=0  if(p=0){
        p:=1;
    }
    pc=1
    pc=2  if(p=1){
        if(g=free){
            g:=id;
            p:=2;
        }
    }
    pc=3
    pc=4
    pc=5
    pc=6  if(p=2){
        p:=3; g:=free
    }
    pc=7
    pc=8  if(p=3){
        p:=0;
    }
    pc=9
    pc=10 }
```

H train

```
while(*){
    if(p=0){
        p:=1;
    }
    if(p=1){
        if(g=free){
            g:=id;
            p:=2;
        }
    }
    if(p=2){
        p:=3; g:=free
    }
    if(p=3){
        p:=0;
    }
}
```

V train
Partial Order Reduction

Key idea:
- The order of independent actions on different threads does not matter
- Note: what is considered independent depends on the property $F \neg p$
Ample set

On state $s_1$, the transitions to $s_2$ and $s_3$ are both enabled.

- $\text{enabled}(s_1)$

We only want to explore a subset of the enabled set
- $\text{ample}(s_1) \subseteq \text{enabled}(s_1)$
We have 3 goals in computing *ample(s)*

- Using ample instead of enabled should give us a much smaller graph
- Using ample instead of enabled should still allow us to find what we are looking for
- Computing ample should be easy
Independence and Invisibility

Independence:
- Actions $a$ and $b$ are independent iff:
  - $a$ does not disable $b$ and vice-versa
  - Commutativity: $a(b(s)) = b(a(s))$

Invisibility:
- $a$ and $b$ should not affect the values of any relevant property
Ample is computed heuristically

Computing it precisely is too hard, but we can find actions that are definitely not in ample(s) and can therefore be ignored.

What we need to consider:
- Actions that share variables with the property
- If two actions share variables, they are dependent
- If two actions appear in the same thread they are dependent