The following problems explore proof techniques for showing that one module implements another module. There are two problems in this problem set; please turn in each problem on a separate sheet of paper. Also give the amount of time you spend on each problem.

**Problem 1. Turtle Robot**

A turtle robot can move freely in a two-dimensional plane. The following module specifies robot’s behavior.

```plaintext
MODULE Turtle
EXPORT Move, Position

TYPE Coord = [x: Int, y: Int]
Path = SEQ Coord

VAR p: Path := {}

APROC Move(dx: Int, dy: Int) =
  << p := p + { Coord(x:=dx, y:=dy) } >>

FUNC sumdx() = + : (p * \ coord | coord.x)
FUNC sumdy() = + : (p * \ coord | coord.y)
FUNC Position() -> Cord = Coord(x:=sumdx(), y:=sumdy())
```

The robot is controlled by an embedded processor that has limited memory.

a) Write an implementation TurtleImpl that implements the Turtle specification and avoids storing the entire path of the robot.

b) Prove that your TurtleImpl implements Turtle.

**Problem 2. Lossy Memory**

Consider a memory that sometimes silently refuses to perform a write to a location. The specification of this memory is given by the following module, which is a modification of the Memory module on page 3 of the Handout 5.

```plaintext
MODULE LMemory [A, V] EXPORT Read, Write =

TYPE M = A -> V
VAR m := Init()

APROC Init() -> M = << VAR m’ | (ALL a | m’! a) => RET m’ >>

FUNC Read(a) -> V = << RET m(a) >>
APROC Write(a, v) = << m(a) := v [] SKIP >>
END Memory
```
a) Write a write-back cache implementation LWBCache of LMemory. Assume that a write to the cache always succeeds (provided that the cache is not full), so do not lose data when writing to the cache. Assume further that writing the data back to the underlying memory may result in writes being silently skipped.

b) Prove that your LWBCache implements LMemory using the appropriate proof technique.