6.837 Introduction to Computer Graphics
Quiz 1
Tuesday October 26, 2003 2:40-4pm
One page of notes allowed

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1 Light and shading [ /15]

1.1 Phong [ /2]
What is the visual effect of the Phong exponent? Write one or two sentences.

1.2 Lambertian materials [ /4]
For a diffuse material, the BRDF is constant, yet the intensity varies across a rounded surface. Why? Write one or two sentences.

1.3 Falloff [ /2]
In the real world, how fast does light intensity decrease with respect to distance from a point light source?
1.4 Shadows [3]

Which algorithm produced the shadows in the following image: ray-casting, shadow maps or shadow volumes? How can you tell?

1.5 Recursive ray tracing [4]

If the objects in your scene have reflective and transmissive color no “brighter” than (0.5, 0.5, 0.5) and the recursive ray weight cutoff is 0.05, what is the maximum depth of the ray tree in your ray tracer? Explain briefly.
2 Linear algebra [ /12]

2.1 Linearity [ /3]
What formal property defines linearity? Write one or two equations that characterize a linear operator.

2.2 Translation [ /2]
Show that translation is not linear in Euclidean space.

2.3 Translation in homogeneous coordinates [ /2]
Give the 4x4 matrix $M$ for translation by a vector $(t_x, t_y, t_z)$ in homogeneous coordinates.
2.4 Sum in homogeneous coordinates

Consider the coordinate-by-coordinate sum of two homogeneous vectors (four components each). What’s the 3D geometric interpretation of this sum? First treat the case where both vectors have \( w = 1 \), then the more general case where \( w \neq 0 \).

Now show that translation is a linear operation in homogeneous coordinates.

3 Ray-cone intersection

3.1 Cone equation

Give the implicit equation for a double-cone of angle \( \theta \) centered on the origin and oriented along the \( z \) direction.
3.2 Ray equation \[ /2 \]
Give the parametric equation of a ray with origin \( O \) and direction \( \vec{d} \).

3.3 Intersection \[ /6 \]
Give the equation for the parameter \( t \) at the ray-cone intersection. Solve for \( t \). Do you always find a solution? Why or why not?

3.4 Finite cone \[ /3 \]
How do you need to change the equation or the code to handle half-infinite cones (only the half in the positive \( z \) half space)?
4 Rasterization [ /8]

4.1 Efficiency [ /4]

Here is pseudo-code for triangle rasterization.

For each triangle
   Set up coefficients $a_i$, $b_i$, $c_i$ for the 3 edge equations
   For each screen pixel
      For each edge $f_i = a_i x + b_i y + c_i$
      If all $f_i$ are positive
         display pixel

Give two possible optimizations.

4.2 Line rasterization [ /4]

How are line rasterization and ray-intersection acceleration related?

How do they differ?