Admin
New topic—sampling.

MST
Review Background
- kruskal
- boruvka
- verification

Idea: sample representative subproblem.
Intuition: “fences” like selection algorithm.
Sampling theorem:
- Heavy edges
- pick $F$ with probability $p$
- get $n/p$ $F$-heavy edges

Recursive algorithm without boruvka:

$$T(m, n) = T(m/2, n) + O(m) + T(2n, n) = O(m + n \log n)$$

(sloppy on expectation on $T(2n,n)$)

Recursive algorithm with 3 boruvka steps:

$$T(m, n) = T(m/2, n/8) + c_1(m + n) + T(n/4, n/8)$$

$$\leq c(m/2 + n/8) + c_1(m + n) + c(n/4 + n/8)$$

$$= (c/2 + c_1)m + (c/8 + c_1 + c/4 + c/8)n$$

$$= (c/2 + c_1)(m + n)$$

so set $c = 2c_1$ (not sloppy expectation thanks to linearity).

Notes:
- Chazelle $m \log \alpha(m, n)$ via relaxed heap
- Ramachandran and Peti optimal deterministic algorithm (runtime unknown)
- open questions.