Shape Influence in Geodesic Active Contours

Leventon, Grimson, Faugeras

Presenter: Polina Golland

Images courtesy of Michael Leventon.
Segmentation

- Active Contours, ‘Snakes’, Level Sets
Geodesic Active Contours

- Snake methodology defines an energy function $E(C)$ over a curve $C$ as
  \[
  E(C) = \beta \int |C'(q)|^2 dq - \lambda \int |\nabla I(C(q))| dq
  \]

- Caselles, et al. reduced the minimization problem to the expression
  \[
  \min_{C(q)} \int g(|\nabla I(C(q))|) |C'(q)| dq
  \]
  where $g$ is a function of the image gradient of the form $\frac{1}{1 + |\nabla I|^2}$.

- The following curve evolution equation can be derived using Euler-Lagrange.
  \[
  \frac{\partial C(t)}{\partial t} = g \kappa \mathbf{N} - (\nabla g \cdot \mathbf{N}) \mathbf{N}
  \]
  where $\kappa$ is the curvature and $\mathbf{N}$ is the normal.

- By defining an embedding function $u$ of the curve $C$, the update equation for the higher dimensional surface is given by (Osher, Sethian '88):
  \[
  \frac{\partial u}{\partial t} = g \kappa |\nabla u| + \nabla u \cdot \nabla g
  \]
Shape Prior for Segmentation

- Train on a set of shapes
  - Mean shape
  - PCA-based model of variation

- Bias the segmentation towards likely shapes
Training Data

- The training set, $T$, consists of a set of surfaces: $T = \{ u_1, u_2, \ldots, u_n \}$

$$T = \{ \ldots \}$$

- The mean shape

$$\mu = \text{[Image of mean shape]}$$
Principal Modes of Variation
(using PCA)
Shape Distribution
Modified Evolution Equation

\[ u(t + 1) = u(t) + \lambda_1 \left( g \left( c + \kappa \right) |\nabla u(t)| + \nabla u(t) \cdot \nabla g \right) + \lambda_2 \left( u^*(t) - u(t) \right) \]
Shape+Pose Estimation

- Given the current contour

\[ \langle \alpha_{\text{MAP}}, p_{\text{MAP}} \rangle = \arg\max_{\alpha, p} P(\alpha, p \mid u, \nabla I) \]

- Probability model

\[
P(\alpha, p \mid u, \nabla I) = \frac{P(u, \nabla I \mid \alpha, p)P(\alpha, p)}{P(u, \nabla I)} = \frac{P(u \mid \alpha, p)P(\nabla I \mid \alpha, p, u)P(\alpha)P(p)}{P(u, \nabla I)}
\]
Shape+Pose Estimation (cont’d)

\[ P(u \mid \alpha, p)P(\nabla I \mid \alpha, p, u)P(\alpha)P(p) \]

- Inside term
  \[ P(u \mid \alpha, p) = \exp(-V_{outside}) \]

- Gradient Term
  \[ P(\nabla I \mid \hat{u}^*, u) = \exp\left(-\left| h(\hat{u}^*) - |\nabla I| \right|^2 \right) \]

- Shape prior: Gaussian (PCA model)
- Pose prior: uniform over the image
Modified Evolution Equation

\[ u(t + 1) = u(t) + \lambda_1 \left( g (c + \kappa) |\nabla u(t)| + \nabla u(t) \cdot \nabla g \right) \]
\[ + \lambda_2 \left( u^*(t) - u(t) \right) \]
Corpus Callosum Segmentation
Corpus Callosum Segmentation
Spine Modes

- 3D Models of seven thoracic vertebrae (T3-T9)
Spine Mean Shape
Spine 1\textsuperscript{st} Mode of Variation
Segmentation of the Vertebrae
Comparison to human expert

Discrepancy in Segmentations

95% Hausdorff Distance (mm)

Case # (sorted by error)

Automatic Segmentation
Second Manual Segmentation
Summary

• Introduced shape prior into curve evolution
  – Previous work – Fourier decomposition and ASH
  – This one is the first for the level set formulation
• PCA on training examples
• 2D and 3D
• Several follow-up methods that perform optimization differently.