2 ways to represent variables in 3SAT:

1. Dual-rail logic:
   - Variable gadget forces exclusive OR of 2 "semi-wires" (true & false)
   - Semiwire connects to clauses \( \wedge \) variable (active only when chosen)
   (e.g. Nintendo, pushing blocks, Phutball - most 3SAT reductions we've seen)

2. Binary logic: (not just Circuit SAT)
   - Wire gadget has 2 (types of) solutions
   - Split gadget to make copies of wire (e.g. flat-foldable crease patterns)

   - Circuit SAT also needs terminator gadget to start a variable wire

\[ \Rightarrow \text{in both cases, may need} \]
   - Turn gadget to route (semi)wires
   - Crossover gadget to cross (semi)wires
Akari/Light Up: [Nikoli 2001]
- given square grid with some obstacles
- some obstacles have a number
  → how many (0-4) edge-adjacent lights
- light illuminates like rook, up to obstacles
- goal: place lights in blanks so that
  - black space lit
  - no lights light each other
  - satisfy numbers

NP-complete by reduction from Circuit SAT: [McPhail 2005]
- wire, turn gadgets
- split/negation gadget
  → split & negation gadgets (via terminators)
- OR/XNOR gate
- crossover gadget: just XORs!
Minesweeper: given square grid of numbers & unknowns & possibly mines

Consistency: does there exist a solution?
- e.g. see whether mine at \( x \) is consistent with (consistent) info so far: if not, play \( x \)
  \( \rightarrow \) special case of interest

NP-complete by reduction from Circuit SAT
  [Kaye 2000]
  - wire, terminator
  - split/NOT/turn
  - phase changer (shift by 2) via 2 NOTs
  - AND
  - crossover gadget: just use NANDs!
  [Goldschläger 1977]
Winning: can I force a win? (no guessing)
i.e. figure out all squares? [Hearn 2006]

Inference: can I figure out any squares? [Scott, Stege, van Rooij 2011]

$\in \text{CoNP}$: proof of NO = 2 differing solutions

\text{CoNP-complete} by reduction from \text{Circuit UNSAT}:

\[ \neg \exists x_1 \cdots x_n \text{ s.t. } f(x) \]
\[ \equiv \forall x_1 \cdots x_n : \neg f(x) \]

- wire, turn, terminator
- NOT, OR, shifter
- split
- crossover: just use NORs!

- special care to ensure equal # mines in all cases (# mines part of puzzle) & ports aligned (middle of 3)

- unsatisfiable $\iff$ output forced to be F

\text{Planar Circuit SAT}: given noncrossing circuit

- only NAND or - only NOR (\& splitters)
**Candy Crush / Bejeweled**

- given square grid of colors (among 6)
- move = swap two edge-adjacent squares
- whenever 3 equal colors in a row/column: 3 squares disappear & columns fall

(\textit{”pop”})

**NP-complete** to get \( p \) points with \( k \) moves by reduction from 3SAT

... in model where pops happen sequentially bottom to top

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**Claim:** worse to trigger wire (even \( x \& \bar{x} \)) directly

- only use 5 colors

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[Walsh 2014]
NP-complete with simultaneous pops by reduction from 1-in-3SAT
- works for many goals:
  - p points in k moves
  - p points
  - pop p gems
  - p moves
  - pop a specific gem

[Guàlà, Leucci, Natale 2014]