Planar 3SAT:
- NP-hard special case of 3SAT
- variable-clause bipartite graph is planar
  \[ \rightarrow \text{edge} \ (v_i, c_j) \text{ whenever } v_i \text{ or } \overline{v_i} \text{ is in } c_j \]
+ remains planar after connecting variables in a cycle: \[ v_1 \rightarrow v_2 \rightarrow \cdots \rightarrow v_n \rightarrow v_1 \]
  - OR after connecting variables & clauses in a cycle
+ remains planar if we require \( v_i \)'s positive connections separated from negative connections
i.e. split \( v_i \) into \( v_i \) positive connections \( \overline{v_i} \) negative connections
+ remains planar if we require all positive connections on one side of cycle & negative connections on other side \( \Rightarrow \) monotone 3SAT

\[ \text{[de Berg \\& Khosravi - Cocoon 2010]} \]
- reductions from 3SAT
Planar rectilinear 3SAT:
- variable = horizontal segment on x axis
- clause = horizontal segment (off x axis) + 3 vertical connections to variables
- no crossings/overlap (other than connections)

Planar monotone rectilinear 3SAT: as above
+ monotone 3SAT: each clause all positive or all negative
+ positive clauses above x axis
+ negative clauses below x axis

- reduction from planar rectilinear 3SAT

Careful:
- if all clauses on one side of variable cycle (above x axis in planar rectilinear 3SAT) then EP via tree dynamic program

⇒ if clauses also connected in a path then EP (would force clauses on same side)
(wanted this e.g. for Push-1/Nintendo)
Planar 1-in-3SAT: [Dyer & Frieze 1986]
- NP-hard special case of 1-in-3SAT
- variable-clause bipartite graph is planar
  + remains planar after connecting variables in a cycle: \( v_1 \rightarrow v_2 \rightarrow \ldots \rightarrow v_n \rightarrow v_1 \)
  - OR after connecting variables & clauses in a cycle

Reduction from Planar 3SAT:
- clause gadget

Planar positive 1-in-3SAT: no negations [Mulzer & Rote - J.ACM 2008]
  + remains planar after connecting variables in a cycle: \( v_1 \rightarrow v_2 \rightarrow \ldots \rightarrow v_n \rightarrow v_1 \)

Rectilinear ... :
- variable = horizontal segment on x axis
- clause = horizontal segment (off x axis)
  + 3 vertical connections to variables

Reduction from Planar 3SAT:
- equal & not-equal gadgets
- remove negations
- expand clauses (2 cases: \( u=0 \) or \( 1 \))
Careful: Planar NAE 3SAT is polynomial!  
[Moret-SIGACT News 1988]

Reduction to Planar Max Cut: 2-color vertices of planar graph to maximize red-blue edges

\[ \text{NP} \text{ \textbackslashin \ P} \] [Orlova & Dortman 1972] [Hadlock-SICOMP 1975]

(in dual, red-blue edges are non-doubled edges in Chinese Postman problem)

- variable gadget / wire
- NAE clause

Planar X3C:  
[Dyer & Frieze 1986]

- bipartite graph of elements vs. 3-sets is planar
- reduction from planar 1-in-3SAT

Planar 3DM:  
[Dyer & Frieze 1986]

- special case where elements are 3-colored & each 3-set is trichromatic
- remains planar if elements connected in cycle
- reduction from planar 1-in-3SAT
Planar vertex cover: [Lichtenstein 1982]
- given a planar graph
- choose k vertices to hit all edges
- reduction from planar 3SAT
  - variable gadget: even cycle
  - clause gadget: triangle
- maximum degree 3

Planar (directed) Hamiltonian cycle: [Lichtenstein 1982]
- reduction from planar 3SAT
  - visit cycle through variables
  - variable gadget = ladder
  - clause gadget
  - can’t jump var. ⇒ clause ⇒ other var.
- same reduction claimed for undirected

Shakashaka [Guten 2008; Nikoli 2012-]
- reduction from Planar 3SAT

Flattening fixed-angle chains: [Soss & Toussaint 2000]
- reduction from Partition
- reduction from planar monotone rectilinear 3SAT
[Demaine & Eisenstat 2011]