Today: video games & PSPACE. First some NP:

**Metatheorem 1:** [Viglietta-Fun 2012 & arXiv:1201.4995]
- player traversing planar environment from start
- location traversal & single-use paths \( \Rightarrow \) NP-hard
  \>
  player must visit some locations
  \>
  player can traverse only once
- reduction from Planar Max-deg-3 Hamiltonicity
- vertex \( \Rightarrow \) location traversal
  \>
  visit each vertex \( \geq \) once
- edge \( \Rightarrow \) single-use path
  - max. degree 3 \( \Rightarrow \) never revisit vertex
- applications
  - Boulderdash
  - Lode Runner
  - Zelda II

  [Aloupis, Demaine, Guo, Viglietta 2014]

**Metatheorem 2:** [Viglietta-Fun 2012 & arXiv:1201.4995]
- location traversal & tokens + toll roads \( \Rightarrow \) NP-hard
  \>
  can pick (one) up \( \updownarrow \)
  \>
  need token to traverse
- vertex \( \Rightarrow \) location traversal + token
- edge \( \Rightarrow \) toll road
  - traversing twice \( \Rightarrow \) stranded without token
- application: Pac-Man
Recall from L1:

\[ \text{PSPACE} = \{ \text{problems solvable in polynomial space} \} \]
- \( \subseteq \text{EXP} \): only exponentially many states
- \( \supseteq \text{NP} \): simulate all executions, take running or
- \( = \text{NPSPACE} \quad [\text{Savitch 1970}] \)

\underline{Base PSPACE-complete problems:}
- simulate linear-space algorithm (e.g. Turing machine)
- \( \text{QSAT} \quad \text{(AKA QBF & TQBF)} \)
  - given (fully) quantified Boolean formula, is it true?
  - e.g. \( \forall x \exists y : (\overline{x} v y) \land (x v \overline{y}) \quad (x \equiv y) \)
  - can assume quantifiers in front (prenex) & alternate \( \forall / \exists \quad (\exists \text{ only } \Rightarrow \text{SAT} \Rightarrow \text{NP-comp}) \)
- Schaefer-style dichotomy theorem:
  - \( \in \text{P} \Leftrightarrow \text{Horn, dual-Horn, 2-CNF, or X(N)OR} \)
    (not if satisfied by all true/all false)
  - PSPACE-complete otherwise \([\text{Chen-C.Surveys 2009}]\)
- planar Q3SAT \([\text{Schaefer - SIComp 1981}] \quad [L^7] \)
  - add \( \exists \) for new variables at end of quantifiers
- planar 1-in-3 Q3SAT (as in \( L^7 \))
**Metatheorem 3:** [Viglietta - Fun 2012 & arXiv:1201.4995]

- player traversing planar environment from specified start to specified goal
- door + open pressure plate + close pressure plate
  - traversable → walk on it
  - only if open → open specific door → ditto, close
- reduction from Q3SAT
- clause gadget
- existential quantifier gadget
- universal quantifier gadget
- one plate of each type for each door
- applications:
  - many FPSs e.g. Doom, Quake, Heretic, Hexen, ...
  - many RPGs e.g. Eye of the Beholder
  - many adventure games e.g. SCUMM engine
    (Maniac Mansion, Monkey Island, Space Quest, ...)
  - Prince of Persia
Metatheorem 4: buttons instead of pressure plates

- optional: can press or not
- activates 3 doors at once

- pressure plate gadget

- in fact 2 doors per button suffice
  [Bodlaender & van der Zanden - unpublished 2014]

- applications: MANY
  - Sonic the Hedgehog (Sega Genesis)
  - The Lost Vikings (Super NES; PC) "Erik the Swift"
  - Tomb Raider (Sega Saturn & PS1; PC)

Metatheorem 5: [Aloupis, Demaine, Guo, Viglietta 2014]

- door with traverse, open, close paths \(\Rightarrow\) PSPACE-hard
  only if opened can open \(\Rightarrow\) must close

- applications:
  - Legend of Zelda: A Link to the Past
    (Ocarina of Time, Majora's Mask, Oracle of Seasons, The Minish Cap, Twilight Princess \( \Rightarrow \) PushPush-1)
  - Donkey Kong Country 1, 2, 3
  - Super Mario Bros. [Demaine, Viglietta, Williams - unpublished, 2014]
  - Lemmings [Viglietta - Fw 2014]