Recall: (L18)

**Bounded 2-player Constraint Logic (2CL)**
- each edge is either white or black
- each edge can be reversed only once
- **goal**: each player has target edge & wins if they reverse it

- PSPACE-complete for planar constraint graphs with white AND, SPLIT, OR, CHOICE & VARIABLE vertex
  - reduction from impartial game positive CNF SAT
  - players take turns setting variables
  - positive ⇒ white wants true, black wants false
  - black can't win (edge irreversible)
  - white wins ⇔ formula satisfied
  - crossover gadget (only use of CHOICE)
- can make OR protected using free edge
  - no constraint at degree-1 end
Amazons: [Walter Zawalskas 1988]
- queens on chessboard
- \( \text{move} = \text{queen move} + \text{queen shot} \)
  - destroy board position at queen-reachable location
- last player to move wins

- PSPACE-complete [Hearn 2005]
  - polynomial \# moves: shot consumes board
  - reduction from Bounded 2CL
Konane [Hawaii — ancient Hawaiian Polynesians] (documented by Captain James Cook in 1778)

- move = jump your piece over 1 or more opponent pieces in a straight line:
  \[ \bigcirc \rightarrow \bigcirc \rightarrow \bigcirc \rightarrow \bigcirc \]
  \[ \rightarrow \text{remove captured opponent pieces} \]
- last player to move wins

- PSPACE-complete [Hearn 2005]
  - polynomial # moves: move consumes \( \geq 1 \) piece
  - reduction from Bounded 2CL
  - conditional gadget for AND, SPLIT, shift:
    - can traverse input 2 \( \rightarrow \) output 2
      only after input 1 \( \rightarrow \) output 1 \( \text{(else captured)} \)
    - ignore output 1 \( \Rightarrow \) AND
    - prime input 2 \( \Rightarrow \) SPLIT
    - both \( \Rightarrow \) parity shift
Cross Purposes: [Michael Albert 2004]
- black stones = 1x1x2 towers
- white stones = fallen towers
- move = \[ \text{●} \rightarrow \times \text{●●} \] (right)
- Vertical player can only move up/down
- Horizontal player can only move left/right
- last player to move wins

- PSPACE-complete [Hearn 2005]
  - polynomial # moves: move consumes black stone
  - reduction from Bounded 2CL
  - \( H \) forced to help \( V \) after variable settings
  - protected OR (& free edge) to avoid second activation terminating leaving \( H \) w/o move

Stochastic Games: [Papadimitriou - JCSS 1985]
- one player (of 2) plays randomly “nature”
- PSPACE-complete to win with probability \( > \frac{1}{2} \) (via amplification)
- SSAT: \( \exists x_1: \forall x_2: \exists x_3: \forall x_4: \ldots: \Pr\{F\} > \frac{1}{2} \)
- OPEN: real games?
Unbounded formula games: EXPTIME-complete  

[Stockmeyer & Chandra - SICOMP 1979]

- Start with arbitrary variable assignment
- Can set variables to 0 or 1 many times (unlimited)
- All partizan: black & white variables, plus possibly “turn variable” \( t = 0 \) if player 2
  \( t = 1 \) if player 1

\[ G_1: \text{move} = \text{set all variables of your color} \]
& set (common) turn variable \( t = 0 \) if player 2
\( t = 1 \) if player 1
lose if you satisfy (common) 4DNF formula

i.e. move must satisfy common 4CNF formula

\[ G_2: \text{move} = \text{set one variable of your color} \]
(can pass by not changing it)
win if you satisfy your 12DNF formula (2 of them)

\[ G_3: \text{move} = \text{flip one variable of your color} \]
lose if you satisfy your 12DNF formula (2 of them)

\[ G_4: \text{move} = \text{set one variable of your color} \]
(can pass)
win if you satisfy (common) 13DNF formula

\( G_5 = G_6 \) but without CNF constraint

\[ G_6: \text{move} = \text{set one variable of your color} \]
(can pass)
player 1 wins if anyone satisfies (single) CNF formula

Peek: stack of plates with holes; 1 fixed plate
\((G_4)\) black & white plates have 2 states, in & out
- move = manipulate one plate (can pass)
- win if hole all the way through
Membership in EXPTIME = ASPACE [Chandra & Stockmeyer, Kozen - FOCS 1976]

- build set of “mate in k” states for $k = 0, 1, \ldots, c^n$
  
  #moves $\leq$ #states ↑

Unbounded graph games: EXPTIME-complete [Stockmeyer & Chandra - SICOMP 1979]

HAM:
- given simple undirected graph
- each edge black or white & in or out
- move = toggle in/out of an edge of your color
- player 1 wins if in edges form a Hamiltonian cycle (after any move)
- reduction from $G_6$

Block:
- given 3 graphs on the same vertex set
- each player has tokens of their color on some of the vertices ($\leq 1$ token per vertex)
- move = move 1 token of your color along a path in one of the 3 graphs such that target & intermediate vertices have no tokens
- player i wins if they get a token to a vertex $e_i$
- reduction from $G_3$
- variable & clause gadget
Real games that are EXPTIME-complete: \( \Rightarrow \mathcal{F}P \! \)

- Checkers \[\text{[Robson - SIComp 1981]}\]
  - reduction from \( G_3 \) where about to lose after every turn
  - initially players adjust kings between T/F
  - then player mounts an attack: move A or B forcing opponent to follow path, fork as desired
  - if all attack vars. set & no defense vars. set i.e. DNF clause satisfied then get \( x \) free moves
  - with \( x \) free moves can trigger outer spiral \( \Rightarrow \) huge material advantage
  - then can form picket lines \( > \text{size(interior)} \)
    \( \Rightarrow \) win \[\text{[Fraenkel, Garey, Johnson, Schaefer, Yesha - FOCS 1978]}\]

- Chess \[\text{[Fraenkel & Lichtenstein - JCTA 1981]}\]
  - reduction from \( G_3 \)

- Go with Japanese ko rule \[\text{[Robson - IFIP 1983]}\]
Unbounded 2CL:
- each edge is either white or black
- goal: each player has target edge & wins if they reverse it
- \textsc{EXPTIME}-complete even for planar graphs
  - reduction from \textsc{G6}
  - players flip variables
  - if formula satisfied: white (Player 1) will lock all variables & run formula
  - lock = reverse true or false edge
  - black must respond A (then B, C, D) to prevent white from fast win via F
    \Rightarrow black immobilized during locks
- black's slow win is 1 move longer than formula satisfaction \Rightarrow white can't flip its variables after any locking (no time)
- white slower win prevents black from flipping A early, e.g. instead of flipping a variable
- formula uses path equalizer so all satisfying assignments take same time
- \textsc{NCL} crossover
No-repeat rule: [Robson - MFCS 1984]
lose if ever repeat a past game configuration
$\Rightarrow G_1 \cdot G_2 \cdot G_3$ become EXPSPACE-complete
as do Chess & Checkers
- OPEN: is Go with superko (no-repeat) EXPSPACE-complete? (as in USA & China)

Conditional no-repeat rule: [Robson - MFCS 1984]
- two special variables $x$ & $y$
- lose if ever repeat a past game configuration
  $\&$ at most 1 of $x$ & $y$ have changed since
$\Rightarrow G_1$ becomes 2EXPSPACE-complete

Private-information games: [Reif - JCSS 1984]
you can see some but not all of opponent’s state
$\Rightarrow G_1$ 5DNF, $G_2$ DNF become 2EXPSPACE-complete
$\Rightarrow$ version of Peek with half of winning holes visible to each player

Blind games: [Reif - JCSS 1984]
player 1’s entire state is hidden from player 2
$\Rightarrow G_2$ DNF becomes EXPSPACE-complete
$\Rightarrow$ version of Peek above

OPEN: Constraint Logic in all these settings