Problem Set 10

Due: In class on Wednesday, May 5. Starred problems are optional.

Problem 10-1. Use any drawing program\(^1\) you please to draw 2-layer VLSI layouts of the following networks:

(a) complete binary tree on 256 leaves,
(b) butterfly on 64 inputs,
(c) 16 × 16 mesh of trees,
(d) 8 × 8 tree of meshes using divide-and-conquer,
(e) 8 × 8 tree of meshes using fold-and-squash.

Wires should be rectilinear on a grid, and your layouts should have asymptotically optimal area.

Problem 10-2. Let \(V\) be the volume of a 3-dimensional layout of a circuit to solve the circular-shifting problem on inputs of size \(n\), and let \(T\) be the worst-case time of the circuit. Prove an analogous result to \(AT^2 = \Omega(n^2)\), but with volume instead of area.

Problem 10-3. * Consider a tree machine consisting of processors interconnected as a complete binary tree. We wish to package such a machine into VLSI chips, where each chip uses as few pins (I/O connections) as possible. Suppose that a chip can hold up to \(M\) processors. Show that a tree machine of size \(N \geq M\) can be assembled from \(O(N/M)\) identical VLSI chips, each having \(O(1)\) pins.

\(^1\)Suggested drawing programs: xfig, dia (Linux); PowerPoint, Illustrator, SmartDraw, Visio (Windows); ClarisDraw (Macintosh). If you’re more ambitious, write programs outputting PostScript, for example, to draw the graphs.