General Layout Strategy

Tree of meshes (not mesh of trees)

\[ T(n) : \]

\[ N = n^2 \log n \]

\[ n^2 \text{ leaves} \]

Area:

\[ S(n) = 2S(n/2) + n \]

\[ = \Theta(n \log n) \]

\[ A(n) = \Theta(n^2 \log^2 n) \]
Fold and squash:

\[ n^2 \times \log n \text{ layers } \Rightarrow \Theta(n^2 \log^2 n) \text{ area.} \]

**Truncated TOM:** \( \text{TOM}(n, k) \) - top \( k \) levels.

Area = \( \Theta(n^2 k^2) \)

**Decomposition trees**

\( T \) is a \( (w_1, w_2, \ldots, w_r) \) decomposition tree for \( G = (V, E) \):

1. Vertices in \( V \) mapped to leaves of \( T \).
2. Edges in \( E \) run through links of \( T \).
3. Number of edges leaving subtree rooted at depth \( i \) is \( \leq w_i \)

For \( 1 < x \leq 2 \), \( G \) has a \( (w, x) \) decomposition tree if it has a \( (w, w/\alpha, w/\alpha^2, \ldots, \alpha w) \) decomposition tree.

A decomposition tree is balanced if all subgraphs at the same depth have same number of vertices to within 2.
Layout strategy

1. Start with \((w, \sqrt{2})\) decomp tree.
2. Balance the decomp tree.
3. Embed the balanced tree in trunc TM.
4. Use trunc TM layout to yield \(O(w^2\log^2 n)\) area layout.

Balancing decomp trees

Warm-up: Necklace with black and white pearls.
How many cuts to divide into 2 sets, each with half the pearls of each color?

2 cuts suffice. Contingency argument.

Lemma: Consider any 2 strings composed of an even \# of black pearls and an even \# of white pearls. By making at most 2 cuts, the pearls can be partitioned into 2 sets, each containing 2 strings, such that each set has \(1/2\) the pearls of each color.

Pf. (Continuity arg.)
**Lemma.** Let $T$ be a CBT drawn with $n$ leaves on a straight line, and consider any set $S$ of $k$ consecutive leaves of $T$. Then, $T$ has a forest $F$ of complete binary subtrees of $T$ such that:

1. $S = \{\text{leaves of } F\}$
2. at most 2 trees of $F$ have any given height.
3. depth of largest tree in $F$ is $\leq \lg k$.

**Proof.** $F$ be forest of maximal CBT's whose leaves lie only in $S$. (1) & (3) follow. Use induction to prove (2). \(\Box\)

**Theorem.** Let $G$ be a graph on $n$ vertices that has a $(w_1, w_2, \ldots, w_r)$ decomp. tree $T$. Then, $G$ has a $(w_1', w_2', \ldots, w_r')$ balanced decomp. tree $T'$, where

$$w_k' = \frac{1}{4} \sum_{k=i}^{r} w_k.$$

**Proof.** Color leaves of $T$: 1 = node of $G$, 0 = empty.

Recursively split B&W leaves evenly. Each stage has $\leq 2$ strings of consec. leaves from $T$, each of which has $\leq 2$ CBT's of a given height.
Total # wires leaving a string
\[ w_i \leq \sum_{k=i}^{r} w_k \]

Corollary: A graph with a \((w, \alpha')\) decomp tree, \(\alpha\) const, has an \((O(w), \alpha)\) balanced decomp tree.

**Proof.** Sum is geometric:
\[ w_i' = 4 \sum_{k=i}^{r} w_k \]
\[ \leq 4 \sum_{k=i}^{r} \frac{w}{\alpha^{k-1}} \]
\[ \leq 4w \frac{\alpha}{\alpha^i (\alpha - 1)} \]

Graph has \((4w/\alpha, \alpha)\) decomp tree.

Next week: Embed in trans TOM \(\Rightarrow\) layout,
Area-universal networks.

«Exam issues>>