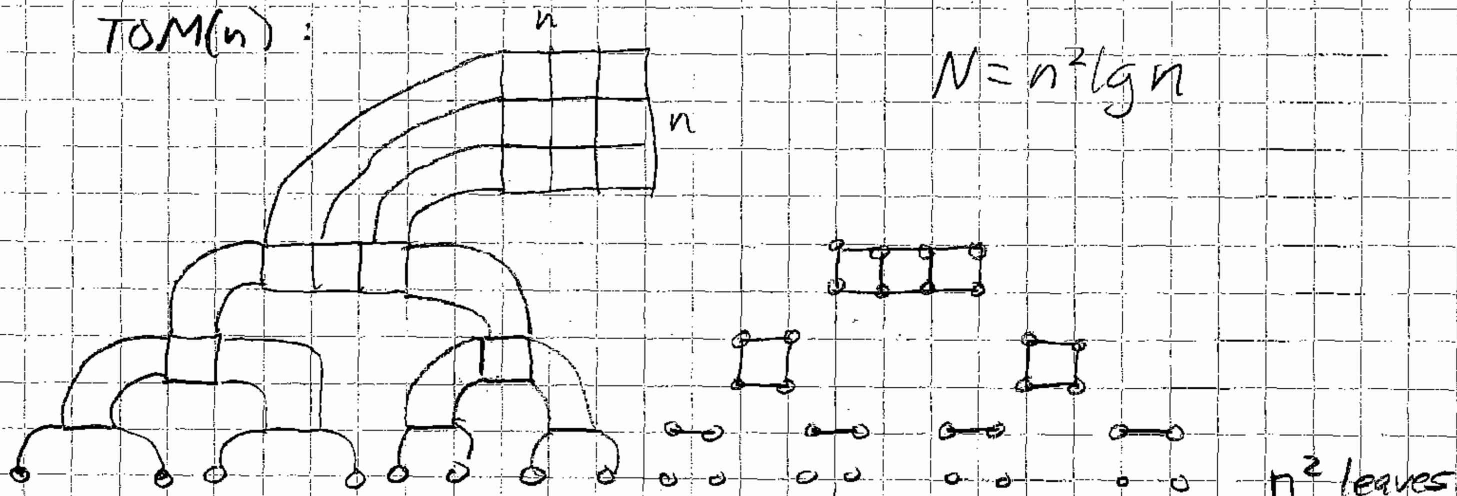
6.896
4/28/04
L20.1

General Layout Strategy

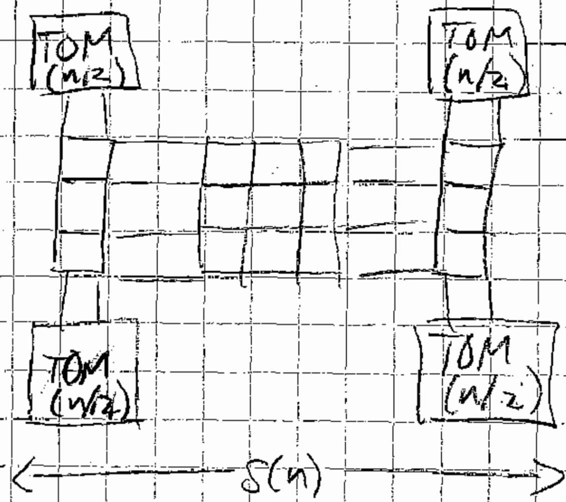
Tree of meshes (not mesh of trees)

TOM(n) :



$N = n^2 \lg n$

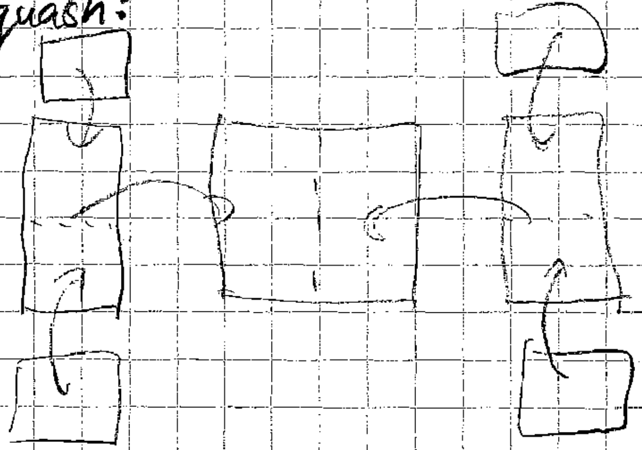
Area:



$S(n) = 2S(n/2) + n$
$= \Theta(n \lg n)$

$A(n) = \Theta(n^2 \lg^2 n)$

Fold and squash:



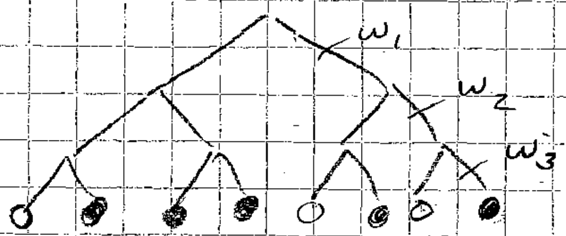
$n^2 \times \lg n$ layers $\xrightarrow{\text{squash}} \Theta(n^2 \lg^2 n)$ area.

Truncated TOM: TOM(n,k) – top k levels.
Area $= \Theta(n^2 k^2)$

Decomposition trees

T is a $\langle w_1, w_2, \ldots, w_r \rangle$ decomposition tree for $G=(V,E)$:

1. Vertices in V mapped to leaves of T.
2. Edges in E run through links of T.
3. #edges leaving subtree rooted at depth i is $\le w_i$



For $1 < \alpha \le 2$, G has a (w, α) decomp tree if it has a $\langle w, w/\alpha, w/\alpha^2, \ldots, O(1) \rangle$ decomp tree.

A decomp tree is balanced if all subgraphs at the same depth have same # vertices to within 1.

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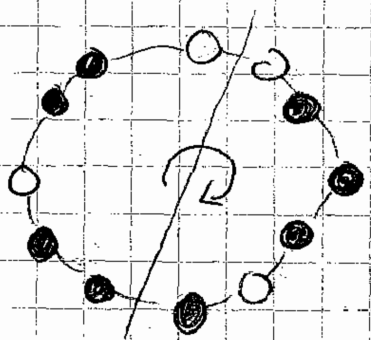
Layout strategy

1. Start with $(w, \sqrt{2})$ decomp tree.
2. Balance the decomp tree
3. Embed the balanced tree in trunc TOM
4. Use trunc TOM layout to yield $O(w^2 \lg^2 n)$ area layout.

Balancing decomp trees

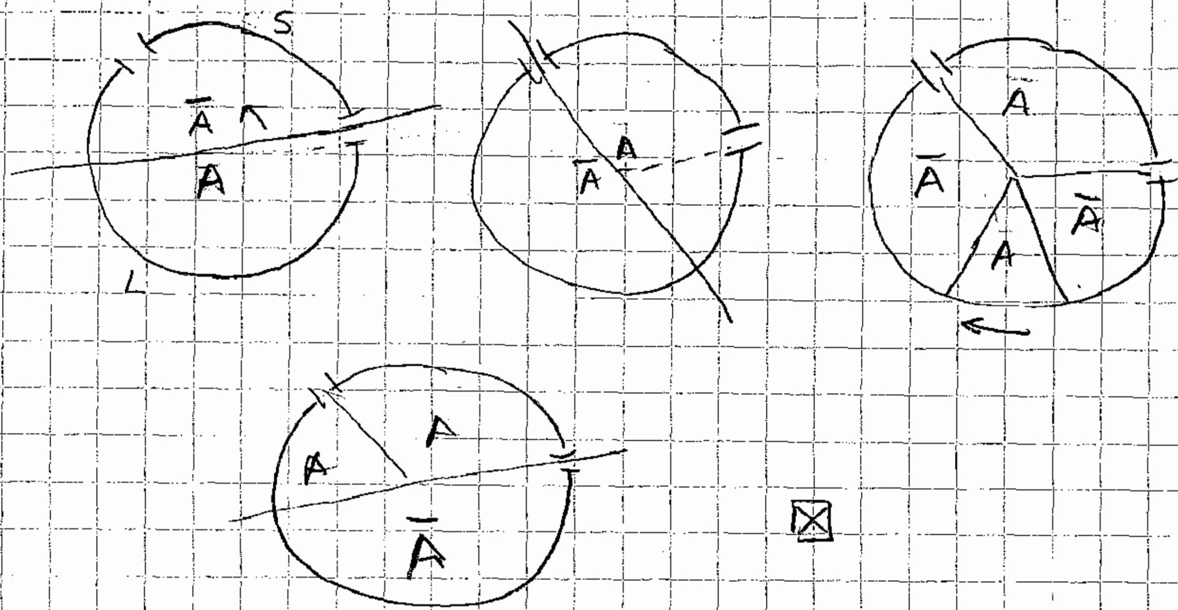
Warm-up: Necklace with black and white pearls.
How many cuts to divide into 2 sets, each with half the pearls of each color?



2 cuts suffice.
Continuity argument.

Lemma. Consider any 2 strings composed of an even # of black pearls and an even # of white pearls. By making at most 2 cuts, the pearls can be partitioned into 2 sets, each containing 2 strings, such that each set has 1/2 the pearls of each color.

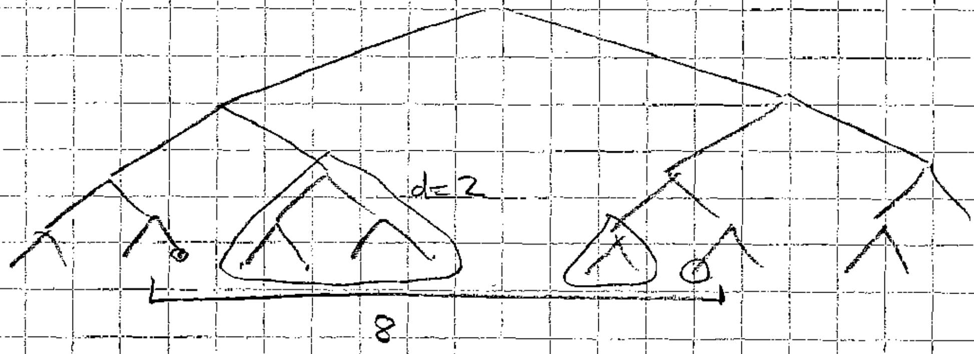
Pf. (Continuity arg.)



∎

Lemma. Let T be cbt drawn with n leaves on a straight line, and consider any set S of k consecutive leaves of T. Then, ∃ a forest F of complete binary subtrees of T ∍

1. S = {leaves of F}
2. at most 2 trees of F have any given height.
3. depth of largest tree in F is ≤ lg k.

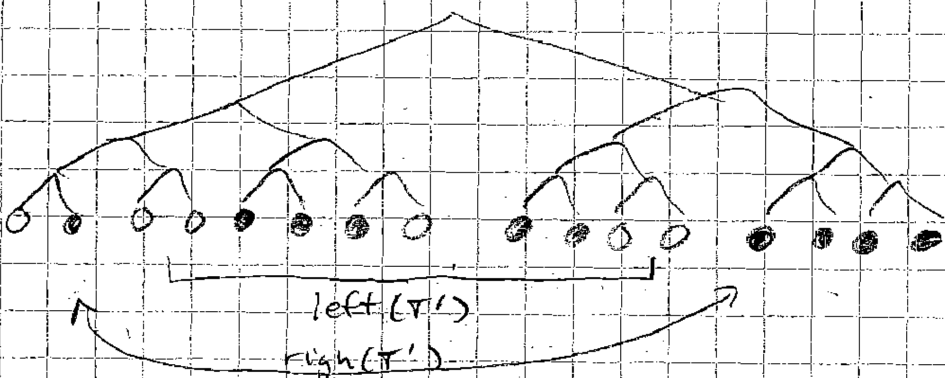


Pf. F be forest of maximal cbt's whose leaves lie only in S. (1) & (3) follow. Use induction to prove (2). ☒

Thm. Let G be a graph on n vertices that has a $\langle w_1, w_2, \ldots, w_r \rangle$ decomp tree T. Then, G has a $\langle w_1', w_2', \ldots, w'_{\lceil \lg n \rceil} \rangle$ balanced decomp tree T', where

$$w_i' = 4 \sum_{k=i}^{r} w_k .$$

Pf. Color leaves of T: 1 = node of G, 0 = empty.



Recursively split B & W leaves evenly. Each stage has ≤ 2 strings of consec. leaves from T, each of which has ≤ 2 cbt's of a given height.

Total # wires leaving a string
$\leq$ sum of wires leaving each of its cbt's.

$$w_i' \leq 4 \sum_{k=i}^{r} w_k. \boxtimes$$

Corollary A graph with a (w, α) decomp tree, α const, has an $(O(w), \alpha)$ balanced decomp tree.

Pf. Sum is geometric:

$$w_i' = 4 \sum_{k=i}^{r} w_k$$

$$\leq 4 \sum_{k=i}^{r} \frac{w}{\alpha^{k-1}}$$

$$\leq \frac{4w}{\alpha^{i-1}} \left(\frac{\alpha}{\alpha - 1}\right).$$

Graph has $(4w\alpha/(\alpha-1), \alpha)$ decomp tree. $\boxtimes$

Next week: Embed in trunc TOM $\Rightarrow$ layout.
Area-universal networks.

«Exam issues»