Problem #1 The purpose of this problem is to develop approximations to the Fermi integral for bulk semiconductors. Consider GaAs \((N_i = 10^6\text{ at }300\text{ K})\) with a donor concentration that varies from \(10^{15}\) to \(10^{19}\) \(\text{cm}^{-3}\).

a. Assuming Boltzmann statistics, calculate and plot the Fermi level as a function of the donor concentration.

b. At high doping density the electron occupancy in the semiconductor is nearly metallic – the Fermi level is in the conduction band and the Fermi-Dirac function can be approximated to be at \(T=0\). In this limit – known as the Sommerfeld approximation, show that the carrier density varies in proportion to 
\[
N \sim (E_F - E_C/kT)^{3/2}.
\]

c. Numerically estimate the Fermi-Dirac integral. Using numerical integration, determine the range of doping where the Fermi level calculation is accurate to 1% under the Boltzmann approximation and the doping range for 1% accuracy under the Sommerfeld approximation.

d. A semiconductor is said to be degenerate when the Fermi level lies within either the conduction or valence bands. At what doping density is this n-type material degenerate?

e. Determine the Fermi level for GaAs doped with acceptors instead of donors. Include the effects of the valence band degeneracy between light and heavy holes, but not the slit-off band. At what doping density is the GaAs degenerate?
Problem #2 The purpose of this problem is to familiarize you with the calculation of carrier concentrations out of equilibrium. A slab of GaAs is illuminated by a beam of light with a wavelength of 519 nm. At this wavelength, the absorption coefficient of GaAs is $\alpha = 10^4$ cm$^{-1}$. The excess carrier lifetimes are $\tau_p = \tau_n = 1$ ns and the slab is much thicker than $1/\alpha$.

a. Determine the excess electron distribution ($\Delta N(x)$) for incident optical powers of $P=1$, 10, and 100 mW in a beam area of 50 $\mu$m$^2$. Neglect carrier diffusion and plot only for 0-5 $\mu$m deep into the sample.

b. Relate the excess carrier concentration to the electron quasi-Fermi level, $E_F(x)$. Plot $E_F(x)$ for the various optical powers again from 0-5 $\mu$m.

c. Since the slab is much thicker than $1/\alpha$, the total photogenerated carrier concentration can be determined. If a DC electric field is applied along the length ($L$) of the slab, determine the photocurrent that flows as a result of the applied field and the incident light.

d. This type of photodetector is known as a photoconductive detector. From (c), it can be seen that the total current can be greater than $qP/h\nu$ - which is the current if every photon generates one electron. Since there is excess photocurrent, there must be electrical gain. What is the physical origin of this gain?
Problem #3 The purpose of this problem is to familiarize you with SimWindows. Consider a p-i-n diode consisting of doped Al$_{0.2}$Ga$_{0.8}$As layers, that are each 1 $\mu$m thick, surrounding an intrinsic (unintentionally doped) GaAs that is 0.5 $\mu$m thick. Consider that each of the Al$_{0.2}$Ga$_{0.8}$As regions are doped with acceptors/donors at $5 \times 10^{17}$ cm$^{-3}$. Use SimWindows to generate all of the data for this exercise.

\[
\begin{array}{cccc}
\text{Al}_{0.2}\text{Ga}_{0.8}\text{As} & \text{GaAs} & \text{Al}_{0.2}\text{Ga}_{0.8}\text{As} \\
\text{p} & \text{i} & \text{n} \\
1 \mu\text{m} & 0.5 \mu\text{m} & 1 \mu\text{m}
\end{array}
\]

a. Write out the Device File that describes this device.

b. Plot the bandgap, electron effective mass, and dielectric constant as a function of position.

c. Plot the bandedge diagrams and quasi-Fermi levels as a function of position for an applied bias of $V=0$, $V=+0.5$ Volts and $V=-2$ Volts.

d. On a log scale, plot the electron and hole density as a function of position for an applied bias of $V=0$, $V=+0.5$ Volts and $V=-2$ Volts.

e. Plot the I-V for this diode. Use a the maximum forward bias that corresponds to current densities of 1 kA/cm$^2$ and a maximum reverse bias of $V=-5$ Volts.

f. For a forward biased current of 1 kA/cm$^2$, how much of the electron current that is injected into the GaAs from the n-type region recombines in the GaAs?
This document is intended as a MATLAB® tutorial for first time users and as a relevant example for more experienced MATLAB® users.

Below is an example of a MATLAB® implementation of the Fermi-Dirac integral and its inverse. The Fermi-Dirac integral is numerically evaluated using the ‘quad8’ function. Evaluating the inverse of the Fermi-Dirac integral is a bit tricky. This algorithm implements a search to look for the Fermi-level that corresponds to the appropriate integral.

function fd = fermi(y,v)
    % fd = fermi(y,v);
    %
    % This is the integrand of the Fermi-Dirac integral from
    % p. 416 of C&C.
    %
    % see also FERMIDIRACINT.M

    % This is a simple example of a function in MATLAB. In the first line,
    % function is the key word that lets Matlab know that you are writing
    % a function. The fd on the left hand side of the equal sign is
    % the output argument, what is returned by the function. on the
    % right hand side of the equal sign is the function name, fermi, with
    % the input arguments in parameters. Note how fd, y, and v are used
    % in the function below.
    %
    % To call this function, type:
    % >> blah = fermi(1,3)
    %
    % blah will be equal to sqrt(1)./(1+exp(1-3)).

    fd = sqrt(y)./(1+exp(y-v));
    % note the dot preceeding the divide. This indicates an
    % element by element division for the vectors y and v.
    %
    % In general, a dot before an operator modifies it from a
    % matrix operation to an element by element operation.
function fdi = fermidiracint(v)
% fdi = fermidiracint(v);
% This function returns the Fermi-Dirac integral of order
% 1/2 given in p. 416 of C&C
% see also FERMI.M
% relative and absolute tolerance
tol = [1e-3, 1e-4];  % pass [] to use defaults

% quad8 is some kind of numerical quadrature to approximate
% the integral.  >>help quad8 OR >>type quad8 for more details
% help is the most valuable matlab command.
% type lets you see what the toolbox is doing, another way to
% learn matlab tricks.
%-----------------------------------------------------------------
warning off;  % turns off the warnings

fdi = zeros(size(v));  % initializes fdi to a vector of zeros
% the same size as v

% step through each v and numerically integrate using a numerical
% quadrature.  The first argument is the name of the function that
% returns the integrand.  The second and third are the limits of
% integration.  The third and fourth are the tolerance and the number
% of points for graphical output, the last is a parameter that is
% passed to fermi(integration_variable,parameter);
for I = 1:length(v)
    fdi(I) = quad8('fermi',0,max(40,v(I)*2),[],[],v(I));
% index vectors and matrices with parentheses.
end

warning on;  % turn on warnings.
% you will get warnings about reaching the recursion level limit
% if you leave warnings on.  This is due to the sharp slope of the
% Fermi-Dirac integrand near y=0. compare the result with trapz.
% To check that it is correct.