Problem #1 This problem explores the electrical properties of a quantum well semiconductor optical amplifier. Consider a 8 nm thick GaAs quantum well with Al$_{0.3}$Ga$_{0.7}$As barriers.

The conduction band discontinuity between GaAs and Al$_{0.3}$Ga$_{0.7}$As is 250 meV. The valence band discontinuity between GaAs and Al$_{0.3}$Ga$_{0.7}$As is 120 meV.

The effective mass for electrons in the GaAs quantum well is $m^*_e=0.067$ and the effective mass for the heavy-holes in the quantum well is $m^*_h=0.34$. The bandgap of GaAs is 1.424 eV.

The effective mass for electrons in the Al$_{0.3}$Ga$_{0.7}$As barrier is $m^*_e=0.091$ and the effective mass for the heavy-holes in the Al$_{0.3}$Ga$_{0.7}$As barrier is $m^*_h=0.38$.

a. Calculate the density of states as a function of energy in the conduction and valence band. Plot for an energy range that corresponds to $3kT$ above the ground state energy. Consider the presence of multiple quantum well states. Plot for both conduction and valence bands.

b. In a quantum well, the density of states is independent of energy so the Fermi integral can be performed analytically. Derive this analytic relationship between the carrier density [cm$^{-3}$] and the Fermi level.

c. In a forward biased p-n junction with a quantum well, quasi-neutrality ensures that the electron and hole density remain equal $(N=P)$. Given that the carriers are captured in a quantum well that sits in the center of the depletion region. Derive an expression for the voltage across the junction as a function of the carrier density. Assume only one subband is occupied in both the conduction band and heavy-hole band.

The gain in the quantum well is given by the following expression:
$$g(N) = g_{\text{max}} \left( f_c(E) - f_v(E - h\nu) \right),$$ where $g_{\text{max}} = 7000$ cm$^{-1}$.

d. Determine the transparency carrier density ($N_0$). Transparency is the bias point where the net gain is equal to zero: $g(N_0)=0$. Assume that the photon energy is tuned to the conduction-to-heavy hole transition energy.

e. Plot the gain ($g(N)$) as a function of carrier density ($N$).

f. Fit the gain to an approximate logarithmic model $g(N) = g_o \ln(N/N_o)$ and determine the fitting parameters $g_o$ and $N_o$.

g. Plot the population inversion factor. Plot over a range of carrier density from half of transparency to twice the transparency carrier density.
Problem #2 This problem explores gain saturation in an optical amplifier and its application to optical switching. Consider a GaAs quantum well semiconductor optical amplifier with the following properties:

- Material gain: \( g(N) = g_o \ln(N/N_o) \) where \( g_o = 2400 \text{ cm}^{-1} \) & \( N_o = 2.6 \times 10^{18} \text{ cm}^{-3} \)
- Optical confinement factor: \( \Gamma = 0.3 \)
- Injection efficiency: \( \eta_i = 0.9 \)
- Volume of active region: \( V_{ac} = 1 \times 10^{-10} \text{ cm}^3 \)
- Group index: \( n_g = 4 \)
- Bimolecular recombination: \( B = 0.8 \times 10^{-10} \text{ cm}^3/\text{s} \)
- Trap assisted recombination: \( A = 10^8 \text{ 1/s} \)
- Auger recombination coeff.: \( C = 3.5 \times 10^{30} \text{ cm}^6/\text{s} \)

a. Near transparency determine the effective recombination lifetime of the semiconductor (when there are no incident photons). What is the dominant contribution to the carrier recombination rate near transparency?

b. Near transparency determine the linearized gain parameters, \( a \) and \( N_o \), where the linearized gain model is \( g(N) = a (N-N_o) \).

c. Using the linearized gain parameters and recombination rate, use the carrier density rate equation to calculate the saturation photon density, \( N_{psat} \):

\[
g(N, N_{psat}) = \frac{g(N, N_{psat}=0)}{1 + N_{psat}/N_{psat}}
\]

d. Since the semiconductor easily saturates, it can be used as a nonlinear material. Consider that the SOA is biased at \( I = 30 \text{ mA} \) and that a digitally modulated beam of light is incident on the SOA. A ‘1’ corresponds to \( N_p = 10 \, N_{psat} \) and a ‘0’ corresponds to \( N_p = 0.1 \, N_{psat} \). Calculate the gain in when there is a ‘1’ at the input and when there is a ‘0’ at the input.

e. If a second beam of light is simultaneously incident on the sample \( P_2 = 0.1 \text{ mW} \) \((N_{p2} << N_{psat})\), it will experience a time varying gain. If the SOA is 500 \( \mu \text{m} \) long, calculate the intensity of the second beam as it exits the SOA when the control beam is ‘1’ and when it is ‘0’. Assume both beams have the same confinement factor.

Problem #3 Background information for the term project.

a. Discuss the application of your device. How and why is it used? What are the key performance metrics for this device?

b. What are the competing approaches to realize the same function served by your device? What are the potential advantages to your device?

c. List three journal articles (published in the last three years) on your device structure. Summarize the results for one of the papers.