Problem #1 This problem explores the spontaneous emission rates in semiconductor materials. Consider a bulk Al$_{0.3}$Ga$_{0.7}$As active region. Reading Chapter 4.4 in Coldren & Corzine.

The bandgap of the Al$_{0.3}$Ga$_{0.7}$As is 1.8 eV.

The effective mass for electrons in the Al$_{0.3}$Ga$_{0.7}$As barrier is $m_e^*=0.091$ and the effective mass for the heavy-holes in the Al$_{0.3}$Ga$_{0.7}$As barrier is $m_h^*=0.38$.

a. For an applied voltage of $V_A=1.5$ V across the active region, determine the electron and hole concentration in the active region. What are the electron and hole quasi-Fermi levels? Use the second Unger approximation to assist you in this calculation. Do this calculation for both $T=300$ K and $T=100$ K.

b. For a single optical mode with photons of energy 1.81 eV, determine the spontaneous emission rate [cm$^{-3}$ s$^{-1}$].

Now consider the spontaneous emission rate into the continuum of modes with energy near 1.81 eV. The spontaneous emission rate [cm$^{-3}$ s$^{-1}$ eV$^{-1}$] into all of the modes within $\Delta\omega$ of a frequency ($\omega$) is given by:

$$R_{sp} = \frac{(1/h) \rho_o(\omega)}{v_g n_{sp} g(\omega,N)}$$

Eq 4.56 in C & C

Where $\rho_o(\omega)$ is the optical density of states

c. Plot the spontaneous emission spectrum for the device at this bias point for both $T=300$ K and $T=100$ K.

d. From the plot in (c), estimate the total spontaneous emission rate [cm$^{-3}$ s$^{-1}$]. What is the lifetime of an electron-hole pair at this bias?

e. What is the steady-state current necessary to replenish the carriers that recombine via spontaneous emission? Neglect non-radiative recombination and stimulated emission
Problem #2 This problem explores the relative rates of Auger recombination in a semiconductor. Reading Chapter 4.5.3 and Appendix A12 in Coldren & Corzine. Consider the following scattering processes:

CCCH  

CHHH

a. Calculate the threshold energy for the CHHH process.

b. Assuming Boltzmann statistics for the electron and hole distributions. Compare the relative rates of Auger scattering by the CCCH process and the CHHH process. Assume the material parameters for GaAs.
Problem #3 This problem is to introduce the method of graphical solution for the effective index of a waveguide. Consider an asymmetric waveguide constructed from a slab of GaAs sandwiched in between semi-infinite slabs of AlAs and Al$_x$Ga$_{1-x}$As. Assume a wavelength in free space of $\lambda = 980$ nm. Reading Appendix A3.2 in Coldren & Corzine.

\[
\begin{array}{c}
\text{Al}_{0.4}\text{Ga}_{0.6}\text{As} \\
\text{GaAs} \quad 0.6\mu\text{m} \\
\text{AlAs, } n = 2.96
\end{array}
\]

For the indices of refraction, use: $n = 2.96$ for AlAs and the formulas on pp 249 in Chuang.

a. Consider a waveguide constructed with one cladding having an aluminum mole fraction of $x = 0.4$ and a waveguide core with a thickness of 0.6 $\mu$m. Determine the normalized propagation parameters: $V$, $a$ and $b$ as defined in Appendix 3 of Coldren & Corzine. From these parameters calculate the propagation wavevector ($\beta$) using Fig A3.2.

b. Determine the ray angle, relative to the plane of the waveguide, that makes-up the fundamental TE guided mode in the waveguide described in part (a).

c. Calculate the overlap for the fundamental mode with the core of the waveguide.

d. Calculate the confinement factor for the second order mode with the core of the waveguide.

e. For a waveguide with a core thickness of 0.6 $\mu$m, what is the range of aluminum mole fraction in the Al$_x$Ga$_{1-x}$As for which the waveguide has only a single guided mode. Make use of the graphical solutions presented.

f. Derive the characteristic expression for the effective index for TM modes. This equation should be analogous to equation A3.10 in Coldren & Corzine. Assume a symmetric index profile for the waveguide.