Introduction to Engineering Systems, ESD.00

System Dynamics

Lecture 3

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From Last Time: Systems Thinking

- “we can’t do just one thing” – things are interconnected and our actions have numerous effects that we often do not anticipate or realize.

- Many times our policies and efforts aimed towards some objective fail to produce the desired outcomes, rather we often make matters worse.

- *Systems Thinking* involves holistic consideration of our actions.

Ref: Figure 1-4, J. Sterman, Business Dynamics: Systems Thinking and Modeling for a complex world, McGraw Hill, 2000
Dynamic Complexity

- **Dynamic** (changing over time)
- **Governed by feedback** (actions feedback on themselves)
- **Nonlinear** (effect is rarely proportional to cause, and what happens locally often doesn’t apply in distant regions)
- **History-dependent** (taking one road often precludes taking others and determines your destination, you can’t unscramble an egg)
- **Adaptive** (the capabilities and decision rules of agents in complex systems change over time)
- **Counterintuitive** (cause and effect are distant in time and space)
- **Policy resistant** (many seemingly obvious solutions to problems fail or actually worsen the situation)
- **Characterized by trade-offs** (the long run is often different from the short-run response, due to time delays. High leverage policies often cause worse-before-better behavior while low leverage policies often generate transitory improvement before the problem grows worse.)
Modes of Behavior

Exponential Growth

Goal Seeking

S-shaped Growth

Oscillation

Growth with Overshoot

Overshoot and Collapse

Image by MIT OpenCourseWare.

Ref: Figure 4-1, J. Sterman, Business Dynamics: Systems Thinking and Modeling for a complex world, McGraw Hill, 2000
Exponential Growth

- Arises from positive (self-reinforcing) feedback.
- In pure exponential growth the state of the system doubles in a fixed period of time.
  - Same amount of time to grow from 1 to 2, and from 1 billion to 2 billion!
- Self-reinforcing feedback can be a declining loop as well (e.g. stock prices)
- Common example: compound interest, population growth

Ref: Figure 4-2, J. Sterman, Business Dynamics: Systems Thinking and Modeling for a complex world, McGraw Hill, 2000
Exponential Growth: Examples

CPU Transistor Counts 1971-2008 & Moore's Law

Curve shows 'Moore's Law'; Transistor count doubling every two years

Ref: wikipedia

Image by MIT OpenCourseWare.
Some Positive Feedbacks underlying Moore’s Law

Goal Seeking

- Negative loops seek balance, and equilibrium, and try to bring the system to a desired state (goal).
- Positive loops reinforce change, while negative loops counteract change or disturbances.
- Negative loops have a process to compare desired state to current state and take corrective action.
- Pure exponential decay is characterized by its half life – the time it takes for half the remaining gap to be eliminated.

Ref: Figure 4-4, J. Sterman, Business Dynamics: Systems Thinking and Modeling for a complex world, McGraw Hill, 2000
Oscillation

- This is the third fundamental mode of behavior.

- It is caused by goal-seeking behavior, but results from constant ‘over-shoots’ and ‘under-shoots’

- The over-shoots and under-shoots result due to time delays - the corrective action continues to execute even when system reaches desired state giving rise to the oscillations.

Ref: Figure 4-6, J. Sterman, Business Dynamics: Systems Thinking and Modeling for a complex world, McGraw Hill, 2000
Interpreting Behavior

• Connection between structure and behavior helps in generating hypotheses

• If exponential growth is observed -> some reinforcing feedback loop is dominant over the time horizon of behavior

• If oscillations are observed, think of time delays and goal-seeking behavior.

• Past data shows historical behavior, the future maybe different. Dormant underlying structures may emerge in the future and change the ‘mode’

• It is useful to think what future ‘modes’ can be, how to plan and manage them

• Exponential growth gets limited by negative loops kicking in/becoming dominant later on
Limits of Causal Loop Diagrams

• Causal loop diagrams (CLDs) help
  – in capturing mental models, and
  – showing interdependencies and
  – feedback processes.

• CLDs cannot
  – capture accumulations (stocks) and flows
  – help in determining detailed dynamics

Stocks, Flows and Feedback are central concepts in System Dynamics
Stocks

- Stocks are accumulations, aggregations, summations over time
- Stocks characterize/describe the state of the system
- Stocks change with inflows and outflows
- Stocks provide memory and give inertia by accumulating past inflows; they are the sources of delays.
- Stocks, by accumulating flows, decouple the inflows and outflows of a system and cause variations such as oscillations over time.
Mathematics of Stocks

- Stock and flow diagramming were based on a hydraulic metaphor

- Stocks integrate their flows:

\[
\text{Stock} (t) = \int_{t_0}^{t} [\text{Inflow} (s) - \text{Outflow} (s)] \, ds + \text{Stock} (t_0)
\]

- The net flow is rate of change of stock:

\[
\frac{d(\text{Stock})}{dt} = \text{Net Change in Stock} = \text{Inflow} (t) - \text{Outflow} (t)
\]

Ref: Figure 6-2, J. Sterman, Business Dynamics: Systems Thinking and Modeling for a complex world, McGraw Hill, 2000
## Stocks and Flows Examples

<table>
<thead>
<tr>
<th>Field</th>
<th>Stocks</th>
<th>Flows</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mathematics, Physics and Engineering</td>
<td>Integrals, States, State variables, Stocks</td>
<td>Derivatives, Rates of change, Flows</td>
</tr>
<tr>
<td>Chemistry</td>
<td>Reactants and reaction products</td>
<td>Reaction Rates</td>
</tr>
<tr>
<td>Manufacturing</td>
<td>Buffers, Inventories</td>
<td>Throughput</td>
</tr>
<tr>
<td>Economics</td>
<td>Levels</td>
<td>Rates</td>
</tr>
<tr>
<td>Accounting</td>
<td>Stocks, Balance sheet items</td>
<td>Flows, Cash flow or Income statement items</td>
</tr>
</tbody>
</table>

**Snapshot Test:**
Freeze the system in time – things that are measurable in the snapshot are stocks.

Example

Ref: Figure 7-2, J. Sterman, Business Dynamics: Systems Thinking and Modeling for a complex world, McGraw Hill, 2000
Example

Ref: Figure 7-4, J. Sterman, Business Dynamics: Systems Thinking and Modeling for a complex world, McGraw Hill, 2000

Image by MIT OpenCourseWare.
Flow Rates

• Model systems as networks of stocks and flows linked by information feedbacks from the stocks to the rates.

• Rates can be influenced by stocks, other constants (variables that change very slowly) and exogenous variables (variables outside the scope of the model).

• Stocks only change via inflows and outflows.
Auxiliary Variables

- Auxiliary variables are neither stocks nor flows, but intermediate concepts for clarity
- Add enough structure to make polarities clear

Ref: Figure 8-?, J. Sterman, Business Dynamics: Systems Thinking and Modeling for a complex world, McGraw Hill, 2000
Aggregation and Boundaries - I

- Identify main stocks in the system and then flows that alter them.
- Choose a level of aggregation and boundaries for the system.
- Aggregation is number of internal stocks chosen.
- Boundaries show how far upstream and downstream (of the flow) the system is modeled.
Aggregation and Boundaries - II

• One can ‘challenge the clouds’, i.e. make previous sources or sinks explicit.
• We can disaggregate our stocks further to capture additional dynamics.
• Stocks with short ‘residence time’ relative to the modeled time horizon can be lumped together.
• Level of aggregation depends on purpose of model.
• It is better to start simple and then add details.
From Structure to Behavior

- The underlying structure of the system defines the time-based behavior.

- Consider the simplest case: the state of the system is affected by its rate of change.

Ref: Figure 8-1, & 8-2 J. Sterman, Business Dynamics: Systems Thinking and Modeling for a complex world, McGraw Hill, 2000

Images by MIT OpenCourseWare.
Population Growth

- Consider the population Model:

- The mathematical representation of this structure is:

$$\text{Net birth rate} = \text{fractional birth rate} \times \text{population}$$

Ref: Figure 8-2, J. Sterman, Business Dynamics: Systems Thinking and Modeling for a complex world, McGraw Hill, 2000

Note: units of ‘b’, fractional growth rate are 1/time
Phase-Plots for Exponential Growth

• Phase plot is a graph of system state vs. rate of change of state

• Phase plot of a first-order, linear positive feedback system is a straight line

• If the state of the system is zero, the rate of change is also zero

• The origin however is an unstable equilibrium.

Ref: Figure 8-3, J. Sterman, Business Dynamics: Systems Thinking and Modeling for a complex world, McGraw Hill, 2000
Time Plots

• Fractional growth rate $g = 0.7\%$/time unit

• Initial state $s_0 = 1$.

• State doubles every 100 time units

• Every time state of the system doubles, so too does the absolute rate of increase

Ref: Figure 8-4, J. Sterman, Business Dynamics: Systems Thinking and Modeling for a complex world, McGraw Hill, 2000
Rule of 70

- Exponential growth is one of the most powerful processes.
- The rate of increase grows as the state of the system grows.
- It has the remarkable property that the state of the system doubles in fixed period of time.
- If the doubling time is say 100 time units, it will take 100 units to go from 2 to 4, and another 100 units to go from 1000 to 2000 and so on.
- To find doubling time:
Negative Feedback and Exponential Decay

- First-order linear negative feedback systems generate exponential decay

- The net outflow is proportional to the size of the stock

- The solution is given by: \( S(t) = S_0 e^{-dt} \)

- Examples:

Net Inflow = -Net Outflow = -d*S

d: fractional decay rate [1/time]

Reciprocal of d is average lifetime units in stock.

Ref: Figure 8-6, J. Sterman, Business Dynamics: Systems Thinking and Modeling for a complex world, McGraw Hill, 2000
Phase Plot for Exponential Decay

- In the phase-plot, the net rate of change is a straight line with negative slope.

- The origin is a stable equilibrium, a minor perturbation in state $S$ increases the decay rate to bring system back to zero – deviations from the equilibrium are self-correcting.

- The goal in exponential decay is implicit and equal to zero.


Image by MIT OpenCourseWare.
Negative Feedback with Explicit Goals

- In general, negative loops have non-zero goals.

- Examples:
  - The corrective action determining net flow to the state of the system is: \( \text{Net Inflow} = f(S, S^*) \)

- Simplest formulation is:
  - \( \text{Net Inflow} = \text{Discrepancy/adjustment time} = (S^*-S)/AT \)

AT: adjustment time is also known as \textit{time constant} for the loop.

Ref: Figure 8-9, J. Sterman, Business Dynamics: Systems Thinking and Modeling for a complex world, McGraw Hill, 2000
Phase Plot for Negative Feedback with Non-Zero Goal

- In the phase-plot, the net rate of change is a straight line with slope $-1/AT$

- The behavior of the negative loop with an explicit goal is also exponential decay, in which the state reaches equilibrium when $S=S^*$

- If the initial state is less than the desired state, the net inflow is positive and the state increases (at a diminishing rate) until $S=S^*$. If the initial state is greater than $S^*$, the net inflow is negative and the state falls until it reaches $S^*$

Ref: Figure 8-10, J. Sterman, Business Dynamics: Systems Thinking and Modeling for a complex world, McGraw Hill, 2000
Half-Lives

- Exponential decay cuts quantity remaining in half in fixed period of time.
- The ‘half-life’ is calculated in similar way as doubling time.
- The system state as a function of time is given by:

\[
S(t) = S^* - \left( S^* - S_0 \right) e^{-t/\tau}
\]

- The exponential term decays from 1 to zero as \( t \) tends to infinity.
- Half life is given by value of time \( t_h \):

\[
t_h = \tau \ln 2
\]
Time Constants and Settling Time

• For a first order, linear system with negative feedback, the system reaches 63% of its steady-state value in one time constant, and reaches 98% of its steady state value in 4 time constants.

• The steady-state is not reached technically in finite time because the rate of adjustment keeps falling as the desired state is approached.

<table>
<thead>
<tr>
<th>Time</th>
<th>Fraction of Initial Gap Remaining</th>
<th>Fraction of Initial Gap Corrected</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$e^{-0} = 1$</td>
<td>1 - 1 = 0</td>
</tr>
<tr>
<td>$\tau$</td>
<td>$e^{-1} = 0.37$</td>
<td>1 - $e^{-1} = 0.63$</td>
</tr>
<tr>
<td>2$\tau$</td>
<td>$e^{-2} = 0.14$</td>
<td>1 - $e^{-2} = 0.87$</td>
</tr>
<tr>
<td>3$\tau$</td>
<td>$e^{-3} = 0.05$</td>
<td>1 - $e^{-3} = 0.95$</td>
</tr>
<tr>
<td>5$\tau$</td>
<td>$e^{-5} = 0.007$</td>
<td>1 - $e^{-5} = 0.993$</td>
</tr>
</tbody>
</table>

Ref: Figure 8-12. J. Sterman, Business Dynamics: Systems Thinking and Modeling for a complex world, McGraw Hill, 2000