Inventory Management
More Probabilistic Demand

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Safety Stock Logic  (given $x_L$, $\sigma_L$, A,D, v, r, & Q)

\[ P[SO] = 1 - CSL \]
\[ SS = k\sigma \]
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\[ SS = k\sigma \]

Eqn 7.20

\[ G(k) = (1-IFR)Q/\sigma \]

\[ E[US] \]

CSL (P₁)

SS

S (ROP)

TRC(s,Q)

CUS (B₂)

CSOO (B₁)

IFR (P₂)

CUS (B₂)
Inputted versus Implied Objectives

Any inputted objective implies all others

Example

- Average demand over time is considered constant
- Forecast of demand is 13,000 units a year ~ iid Normal
- Lead time is 2 weeks
- RMSE of the forecast = 1,316 units per year
- EOQ = 228 units (A=50 $/order, r=10%, v=250 $/item)
- $\sigma_L=258$ units and $\mu_L=500$ units

If mgmt sets $P_1=\text{CSL}=.95$, what is the implied:

- IFR (P2)?
- Cost per Stockout Event (B1)?
- Cost per Item Shorted (B2)?
- What are my expected units short?
An Aside: Lost Sales vs Backorders

So far, we have assumed 100% backorders.

\[ TRSSC(\text{Backorder}) = \nu r (k\sigma_L) + C_{\text{Backorder}} \sigma_L G_u[k] \left( \frac{D}{Q} \right) \]

\[ P[SO] = p_{u\geq}(k) = \frac{Qr}{DC_{\text{Backorder}}} \]

If there are lost sales, then we need to order more each replenishment cycle.

How much? \ldots E[US]

Changes \(G_u[k]\) to \((Q/\sigma)((1-\text{IFR})/\text{IFR})\)

\[ TRSSC(\text{LostSale}) = \nu r \left(k\sigma_L + \sigma_L G_u[k]\right) + C_{\text{LostSale}} \sigma_L G_u[k] \left( \frac{D}{Q} \right) \]

\[ P[SO] = p_{u\geq}(k) = \frac{Qr}{DC_{\text{LostSales}} + Qr} \]
Periodic vs Continuous Review

Suppose that I now must review and order periodically.
- What is my order cost?
- How much should I order if I cannot find Q*?

Convenient transformation of (s,Q) to (R,S)
- (s,Q)= Continuous, order Q when IP≤s
- (R,S)= Periodic, order up to S every R time periods

Allows for the use of all previous decision rules

<table>
<thead>
<tr>
<th>(s,Q)</th>
<th>(R,S)</th>
</tr>
</thead>
<tbody>
<tr>
<td>s</td>
<td>S</td>
</tr>
<tr>
<td>Q</td>
<td>D*R</td>
</tr>
<tr>
<td>L</td>
<td>R+L</td>
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</tbody>
</table>
Periodic vs Continuous Review

- **Same Example**
  - Average demand over time is constant at 13,000 units a year
  - Lead time is 2 weeks
  - RMSE of the forecast = 1,316 units per year
  - EOQ = 228 units (A=50 $/order, r=10%, v=250 $/item)
  - $L=258$ units and $\mu_L=500$ units

- **Find the (R,S) inventory policy and safety stock for $P_2=\text{IFR}=.95$**
  - Review Period= 8 Weeks
  - If this was an (s,Q) policy we would find $G_u[k]$

- **Lets do the same thing with the recommended substitutions:**
  - Q becomes $D*R$ or $(13000)(8/52) = 2000$ units
  - $x_L$ becomes $x_{R+L}=(13000)/(52/10) = 2500$ units
  - $\sigma_L$ becomes $\sigma_{R+L} = 1316/(\sqrt{52/10}) = 577$ units
  - So $G_u[k] = (1-.95)(2000/577) = 0.1733$ giving $k=0.58$
  - $S = x_{R+L} + k\sigma_{R+L} = 2500 + (.58)(577) = 2835$ units

- **Policy becomes order up to 2835 units every 8 weeks.**
Questions?
Comments?
Suggestions?