Inventory Management
Special Cases
Probabilistic Demand

Chris Caplice
ESD.260/15.770/1.260 Logistics Systems
Oct 2006
Special Inventory Cases

- Class A items – worth spending more time on
- Class C items – worth spending less time on
- Fashion or Perishable items – worth handling differently
- Indentured items – worth handling differently
What form of inventory policy?

No hard and fast rules, but some rules of thumb

When & how to spend more time to manage A’ inventory

<table>
<thead>
<tr>
<th>Type of Item,</th>
<th>Continuous Review</th>
<th>Periodic Review</th>
</tr>
</thead>
<tbody>
<tr>
<td>A Items</td>
<td>(s, S)</td>
<td>(R, s, S)</td>
</tr>
<tr>
<td>B Items</td>
<td>(s,Q)</td>
<td>(R, S)</td>
</tr>
<tr>
<td>C Items</td>
<td></td>
<td>Manual ~ (R,S)</td>
</tr>
</tbody>
</table>

When & how to spend less time to manage or reduce ‘C’ inventory
## Comparison of Approaches

<table>
<thead>
<tr>
<th></th>
<th>A Items</th>
<th>B Items</th>
<th>C Items</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Type of records</strong></td>
<td>Extensive, Transactional</td>
<td>Moderate</td>
<td>None – use a rule</td>
</tr>
<tr>
<td><strong>Level of Management Reporting</strong></td>
<td>Frequent (Monthly or more)</td>
<td>Infrequently - Aggregated</td>
<td>Only as Aggregate</td>
</tr>
<tr>
<td><strong>Interaction with Demand</strong></td>
<td>Manual input Ascertain predictability Manipulate (pricing etc.)</td>
<td>Modified Forecast (promotions etc.)</td>
<td>Simple Forecast at best</td>
</tr>
<tr>
<td><strong>Interaction with Supply</strong></td>
<td>Actively Manage</td>
<td>Manage by Exception</td>
<td>None</td>
</tr>
<tr>
<td><strong>Initial Deployment</strong></td>
<td>Minimize exposure (high v)</td>
<td>Steady State</td>
<td>Steady State</td>
</tr>
<tr>
<td><strong>Frequency of Policy Review</strong></td>
<td>Very Frequent (monthly or more)</td>
<td>Moderate – Annually or Event Based</td>
<td>Very Infrequent</td>
</tr>
<tr>
<td><strong>Importance of Parameter Precision</strong></td>
<td>Very High – accuracy worthwhile</td>
<td>Moderate – rounding &amp; approximation is ok</td>
<td>Very Low</td>
</tr>
<tr>
<td><strong>Shortage Strategy</strong></td>
<td>Actively manage (confront)</td>
<td>Set service levels &amp; manage by exception</td>
<td>Set &amp; forget service levels</td>
</tr>
<tr>
<td><strong>Demand Distribution</strong></td>
<td>Consider alternatives to Normal as situation fits</td>
<td>Normal</td>
<td>N/A</td>
</tr>
</tbody>
</table>
Managing Class A Inventory

- When does it make sense to spend more time?
  - Tradeoff between complexity and ‘other’ costs
  - Is the savings worth the extra effort?

- Adding precision
  - Finding ‘optimal’ parameters
  - Using more complex policies

$$TC = vD + A \left( \frac{D}{Q} \right) + vr \left( \frac{Q}{2} + k\sigma_L \right) + B_1 \left( \frac{D}{Q} \right) P[SO]$$

$$TC = vD + A \left( \frac{D}{Q} \right) + vr \left( \frac{Q}{2} + k\sigma_L \right) + B_2 v \left( \frac{D}{Q} \right) \sigma_L G_u (k)$$

Dictates whether item is Class A or not
Managing Class A Inventory

Two Types of Class A items:
- Fast moving but cheap (big D little v \( \rightarrow \) Q>1)
- Slow moving but expensive (big v little D \( \rightarrow \) Q=1)

Impacts the probability distribution used
- Fast Movers - Normal Distribution
  - Good enough for B items
  - OK for A items if \( x_L \geq 10 \) or \( x_{L+R} \geq 10 \)
- Slow Movers – Poisson Distribution (& others)
  - More complicated to handle
  - Ok for A items if \( x_L < 10 \) or \( x_{L+R} < 10 \)
Fast Moving A Items

Finding Better \((s, Q)\) Parameters

- Solve for \(k^*\) and \(Q^*\) simultaneously (why?)
- Assume \(\sim\) Normal & \(B_1\) (Cost per Stockout Occasion)

\[
TRC = A \left( \frac{D}{Q} \right) + vr \left( \frac{Q}{2} + k\sigma_L \right) + B_1 \left( \frac{D}{Q} \right) p_{x \geq k}(k)
\]

\[
\frac{\partial TRC}{\partial Q} = 0 \quad \frac{\partial TRC}{\partial k} = 0
\]

\[
\frac{\partial TRC}{\partial Q} = -A \left( \frac{D}{Q^2} \right) + \frac{vr}{2} - B_1 \left( \frac{D}{Q^2} \right) p_{k \geq k}(k) = 0
\]

\[
\frac{\partial TRC}{\partial k} = 0 + vr\sigma_L - B_1 \left( \frac{D}{Q} \right) f_x(k) = 0
\]

Note that:

\[
\frac{\partial p_{k \geq k}(k)}{\partial k} = -f_x(k)
\]

\[
f_x(k) = \frac{e^{-\frac{x^2}{2}}}{\sqrt{2\pi}}
\]
Finding Better \((s, Q)\) Parameters

- End up with two equations
- How do we solve for \((s^*, Q^*)\)?
- Will the new optimal \(Q^*\) be > or < than the EOQ?
- Will the optimal \(k^*\) be > or < than the old \(k\)?
- What is the impact on safety stock? Cycle stock?

\[
Q^* = EOQ \sqrt{1 + \frac{B_1 p_{x \geq (k)}}{A}}
\]

\[
k^* = \sqrt{2 \ln \left( \frac{DB_1}{\sqrt{2\pi Qvr\sigma_L}} \right)}
\]
Fast Moving A Items

Establish an \((s,S)\) policy
- If \(IP<s\) then order up to \(S\) items (\(=S-IP\))
- More complicated due to ‘undershoots’
- See SPP Section 8.5

Establish an \((R,s,S)\) policy
- Every \(R\) time units, if \(IP<s\) then order up to \(S\) items (\(=S-IP\))
- Even more complicated – but can be programmed
- See SPP Section 8.6
Normal distribution may not make sense – why?

Poisson distribution
- Probability of \( x \) events occurring within a time period
- Mean = Variance = \( \lambda \)

\[
p_k(x_0) = \frac{e^{-\lambda} \lambda^{x_0}}{x_0!} \quad \text{for} \quad x_0 = 0, 1, 2, \ldots
\]

\[
p_{k\leq}(x_0) = \sum_{k=0}^{x_0} \frac{e^{-\lambda} \lambda^k}{k!}
\]

In Excel:
- \( p_k(x_0) = \text{POISSON}(x_0, \lambda, 0) \)
- \( p_{k\leq}(x_0) = \text{POISSON}(x_0, \lambda, 1) \)
Example

Suppose demand $\sim P(\lambda=0.8)$ items per week. We want to set an (R,S) policy for an IFR=.90 where R=1 wk.

We know that

1. $\text{IFR} = 1 - (E[\text{US}] / E[\text{Demand in Period}]) = 1 - (E[\text{US}] / \lambda)$
2. $E[\text{US}] = (1-\text{IFR}) \lambda = (1-.90)(.8) = 0.08$ units

How do I find an S so that $E[\text{US}] \leq 0.08$?

<table>
<thead>
<tr>
<th>x</th>
<th>P[x]</th>
<th>P[x]</th>
<th>F[x]</th>
<th>L[x]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>44.9%</td>
<td>44.9%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>35.9%</td>
<td>80.9%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>14.4%</td>
<td>95.3%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>3.8%</td>
<td>99.1%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>0.8%</td>
<td>99.9%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>0.1%</td>
<td>100.0%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>0.0%</td>
<td>0.0%</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Loss Function for Discrete Function

We find the loss function, \( L(X_i) \), for each value of \( X \) given the cumulative probability \( F(X_i) \).

Start with first value
- \( L(X_1) = \text{mean} - X_1 \)
- \( L(X_2) = L(X_1) - (X_2 - X_1)(1-F(X_1)) \)
- \( L(X_3) = L(X_2) - (X_3 - X_2)(1-F(X_2)) \)
- . . . .
- \( L(X_i) = L(X_{i-1}) - (X_i - X_{i-1})(1-F(X_{i-1})) \)

So, set \( S=2 \) since \( L(2)=0.06 \)
- Policy is order up to 2 units every week

More methods in SPP Section 8.3

<table>
<thead>
<tr>
<th>( x )</th>
<th>( P[x] )</th>
<th>( F[x] )</th>
<th>( L[x] )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>44.9%</td>
<td>44.9%</td>
<td>0.80</td>
</tr>
<tr>
<td>1</td>
<td>35.9%</td>
<td>80.9%</td>
<td>0.25</td>
</tr>
<tr>
<td>2</td>
<td>14.4%</td>
<td>95.3%</td>
<td>0.06</td>
</tr>
<tr>
<td>3</td>
<td>3.8%</td>
<td>99.1%</td>
<td>0.01</td>
</tr>
<tr>
<td>4</td>
<td>0.8%</td>
<td>99.9%</td>
<td>0.009</td>
</tr>
<tr>
<td>5</td>
<td>0.1%</td>
<td>100.0%</td>
<td>0.0088</td>
</tr>
<tr>
<td>6</td>
<td>0.0%</td>
<td>0.0%</td>
<td>0.00878</td>
</tr>
</tbody>
</table>
Managing “C” Inventory

- Establish simple reorder rules
  - Periodic rather than continuous
  - Set for all C items collectively (if possible)
  - Look to reduce the number of order cycles

- Identify & Dispose of Dead Inventory
  - Which items to dispose?
    - Look at DOS (days of supply) for each item = IOH/D
    - Consider getting rid of items that have DOS > x years
  - How much to get rid of?
    - Decision rule: IOH − EOQ − D(v-salvage)/(vr)
  - What do you do with it?
  - When can you not never get rid of C or D or FF items?
Managing “C” Inventory

To Stock or Not to Stock?

- Buy-to-order versus buy-to-stock decision
- Factors
  - System cost for stocking an item
  - Variable cost differential for buy-to-order vs buy-to-stock
  - Cost of temporary backorder
- Decision Rule in SPP Section 9.5
  - Essentially trade off between cost to order and frequency of demand
Questions?
Comments?
Suggestions?