Inventory Management
Material Requirements Planning

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Assumptions: Basic MRP Model

- **Demand**
  - Constant vs **Variable**
  - Known vs Random
  - Continuous vs **Discrete**

- **Lead time**
  - Instantaneous
  - **Constant** or **Variable** (deterministic/stochastic)

- **Dependence of items**
  - Independent
  - Correlated
  - **Indentured**

- **Review Time**
  - **Continuous**
  - Periodic

- **Number of Echelons**
  - **One**
  - Multi (>1)

- **Capacity / Resources**
  - **Unlimited**
  - Limited (Constrained)

- **Discounts**
  - **None**
  - All Units or Incremental

- **Excess Demand**
  - **None**
  - All orders are backordered
  - Lost orders
  - Substitution

- **Perishability**
  - **None**
  - Uniform with time

- **Planning Horizon**
  - Single Period
  - **Finite Period**
  - Infinite

- **Number of Items**
  - One
  - **Many**

- **Form of Product**
  - **Single Stage**
  - Multi-Stage
How many components are there?

Image of iPod Shuffle circuitry removed due to copyright restrictions.
Traditional Management

- Purchasing
- Production
- Marketing
Supply Chain Integration

Information / Planning

Inventory Deployment

Material Requirements Planning
Master Production Scheduling
Distribution Requirements Planning
Inventory Management so far . . .

Traditional techniques . . .

- Forecast demand independently for each item based on usage history
- Establish lot sizes independently for each item based on demand forecasts
- Establish safety stocks independently for each item based on forecast errors

Which make the following assumptions . . .

- Demand is "Continuous"  
  [usage occurs in every period]
- Demand is "Uniform"  
  [average usage per period is stable over time]
- Demand is "Random"  
  [usage in any given period is not known in advance]
Cycle Stock with a Fixed Lot Size

A = $500, r = 25%, v = $50, D = 2000 units/yr, $Q^* = 400$ units

Problem: Intermittent Demand
4 production periods, 500 units/period, Demand rate 2000/year
Fixed Lot Size with Intermittent Demand results in . . .

Can we do better?
Another Wrinkle ... Product Indenture

Bicycle Model 1234

- Frame
  - Fork
  - Front fender
  - Rear Fender
  - Sprocket
  - Crank
  - Pedal
  - Chain guard

- Front Wheel
  - Rim
  - Axle
  - Spoke
  - Tire
  - Tube

- Saddle
  - Shaft
  - Seat
  - Cover

- Rear Wheel
  - Rim
  - Axle
  - Spoke
  - Tire
  - Tube
  - Sprocket

- Handlebars
  - Bar
  - Gooseneck
  - Grip

Note that each item, sub-assembly, component etc. might feed into multiple end products
Combined Demand Impacts

Suppose a widget is part of three items

- Product A – 10 items per week – (3 Weeks OH)
- Product B – 5 items per week – (2 Weeks OH)
- Product C – 7 items per week - (4 Weeks OH)

End demand looks like . . .

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Combined Demand Impacts

But if ordered separately – what will widget demand look like?

- Product A – 10 items per week – (3 Weeks OH)
- Product B – 5 items per week – (2 Weeks OH)
- Product C – 7 items per week - (4 Weeks OH)

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Important to synchronize
Push versus Pull Systems

Simple Example

- You make shovels that have 4 parts:
  - Metal Digger
  - Wooden Pole
  - 2 Screws
- Production is 100 shovels per week:
  - Metal part is made in 400 item batches on first 2 days of the month
  - Handles are procured from Pole Co.
  - Assembly occurs during first week of each month
- How should I manage my inventory for screws?
  - A=$0.25, v=$0.01, r=25%
  - D = 800*12=9600 units per year
  - L = 1 week
- What are the values for . . .
  - Q* =
  - x_L =
  - RMSE(L) =
Push versus Pull Systems

What is my policy if I follow a . . .

- Standard EOQ policy?
  - Order \( \sim 1385 \) (~every other month)
  - What would the Inventory On Hand look like?

- Standard \((s,Q)\) policy?
  - So, since \( \sigma_L = 193 \), pick a CSL=95\% \( k=1.64 \)
  - \( s = 185 + (1.64)193 = 502 \) units
  - Order 1385 units when inventory position \( \leq 502 \)

- Standard \((R,S)\) policy?
  - Select a monthly review policy \( (R=4 \) weeks\)
  - \( x_{L+R} = 9600/(52/5) = 923 \) units
  - \( \sigma_{L+R} = 193(\sqrt{5}) = 432 \) units
  - \( S = 923 + (1.64)432 = 1631 \)
  - Order up to 1631 units every 4 weeks

- Other methods?
Material Requirements Planning

Push vs Pull Systems
- Push – MRP
  - “initiates production in anticipation of future demand”
- Pull – JIT
  - “initiates production as a reaction to present demand”

Major Premises
- Inventory control in a production environment
- Many products, many component parts
- Complex product indenture structure
- Production creates "lumpy" demand

Major Concepts
- Dependent demand versus independent demand
- Requirements calculation versus demand forecasting
- Schedule flow versus stockpile assets
- Information replaces inventory
Material Requirements Planning

**Primary Questions**
- What are we going to make? => use forecast
- What does it take to make it? => use res. req’s & BOM
- What do we have? => use inventory records
- What do we need and when? => use mfg schedules

**Information Requirements**
- Master Production Schedule
- Product Indenture Structure
- Inventory Status
- Ordering Data

**MRP Process**
- Requirements Explosion
- Use of Bill of Materials (BOM)
- Net from Gross Requirements
- Requirements Time Phasing
- Planned Order Release
Example: Bike Co.

BOM Explosion

Bicycle

Level 0

Wheel (2)

Level 1

Crank Assembly (1)

Spoke (86)
Tire (1)

Level 2

Sprocket (1)
Crank (2)
Pedal (2)
## Bill of Materials

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### MRP Approach:
1. Start with Level i demand (i=0)
2. Find Gross Requirements (GR) and On Hand (OH) for Level i
3. Find Net Requirements (NR) for Level i+1 (NR=GR-OH)
4. Establish Planned Order Release (POR) for Level i using Level i lead times
5. Set GR for Level i+1 based on POR for Level i
6. Set i = i+1 and go to Step 2
**Objective:**
Have materials ready for having 25 bikes in week 8

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Ordering Plan

What is missing?
Two Issues

How do we handle capacity constraints?

How do we handle uncertainty?
Decision Variables:
- $Q_i$ = Quantity purchased in period $i$
- $Z_i$ = Buy variable = 1 if $Q_i > 0$, =0 o.w.
- $B_i$ = Beginning inventory for period $i$
- $E_i$ = Ending inventory for period $i$

Data:
- $D_i$ = Demand per period, $i = 1,\ldots,n$
- $C_o$ = Ordering Cost
- $C_{hp}$ = Cost to Hold, $$/unit/period$
- $M$ = a very large number….

MILP Model

Objective Function:
- Minimize total relevant costs

Subject To:
- Beginning inventory for period 1 = 0
- Beginning and ending inventories must match
- Conservation of inventory within each period
- Nonnegativity for $Q$, $B$, $E$
- Binary for $Z$
Approach: Optimization (MILP)

Objective Function

\[
\text{Min } TC = \sum_{i=1}^{n} C_O Z_i + \sum_{i=1}^{n} C_{HP} E_i
\]

s.t.

\[
B_i = 0
\]

\[
B_i - E_{i-1} = 0 \quad \forall i = 2, 3, \ldots, n
\]

\[
E_i - B_i - Q_i = -D_i \quad \forall i = 1, 2, \ldots, n
\]

\[
M Z_i - Q_i \geq 0 \quad \forall i = 1, 2, \ldots, n
\]

\[
B_i \geq 0 \quad \forall i = 1, 2, \ldots, n
\]

\[
E_i \geq 0 \quad \forall i = 1, 2, \ldots, n
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\[
Q_i \geq 0 \quad \forall i = 1, 2, \ldots, n
\]

\[
Z_i = \{0, 1\} \quad \forall i = 1, 2, \ldots, n
\]

Beginning & Ending Inventory Constraints

Conservation of Inventory Constraints

Ensures buys occur only if \( Q > 0 \)

Non-Negativity & Binary Constraints
**MRP: Example**

### TOTAL COST
- 10,500.00
- 7,000.00

### END ITEM
| Lead Time | 2 |
| Setup    | $1,000 |
| Holding  | $5.00 |
| Capacity | 1000 |

### Period | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
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### COMPONENT
- 3,500.00

| Lead Time | 2 |
| QPA       | 3 |
| Setup     | $500 |
| Holding   | $1.00 |
| Capacity  | 2000 |

### Period | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
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**Notes:**
- End Item requires 3 of the same Components
- There is a setup cost, holding cost, and (potential) capacity

**Output from End Item Model becomes input to Component Model**

**Lead time in Weeks**

**Quantity Per Assembly**

---

**Image:** Simple MRP Solution

**Text:**

**Notes:**
- End Item requires 3 of the same Components
- There is a setup cost, holding cost, and (potential) capacity
# MRP: Example

**TOTAL COST**

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<tr>
<th>Item</th>
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**Lead Time**

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**Setup**

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**Holding**

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**Capacity**

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**COMPONENT**

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**Lead Time**

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**Holding**

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**Capacity**

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## MRP: Example

### TOTAL COST
- $7,660.00
- $4,300.00

### END ITEM
- Setup: 2
- Holding: 1,000
- Capacity: 5.00
- Lead Time: 1,000

### OPTIMIZE ITEM SCHEDULES

### Component Capacity Constraint Binding

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### COMPONENT
- $3,360.00
- Lead Time: 2
- QPA: 3
- Setup: $500
- Holding: $1,000
- Capacity: 400

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### Introduce Binding Constraint
# MRP: Example

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**Total Cost:** $8,700.00

**Optimize Item Schedules**

[Component capacity constraint "more binding"]

**Note:** Component constraint redefines the End Item schedule.

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**Note:** Tighten binding constraint.
## MRP: Example

### TOTAL COST

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### Notes:
- Solves the End Items and the Components models separately
- What is the impact? insight?
- Who wins? Loses?

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Keep tight constraint
Handling Uncertainty

Safety Stock
- Add to existing stock levels
- Where would this be applied?

Safety Times
- Pad the planned lead times
- Where would this be applied?
Optimal Lead Time Padding

Let:

\( t \) = Delivery Time, a random variable
\( t' \) = Forecasted Delivery Time
\( \sigma \) = Standard Deviation of the Forecast Error
\( T_p \) = Padded Lead time = \( t' + k\sigma \)
\( Q \) = Lot Size in units
\( v \) = Unit Cost
\( r \) = Holding Cost per unit per time period
\( C_d \) = Shortage Cost per time period

\[
TC[T_p] = \sum_{t=0}^{T_p} rvQ(T_p - t)P[t] + \sum_{t=T_p+1}^{\infty} C_d(t - T_p)P[t]
\]

\[
CSL^* = \frac{C_d}{C_d + rvQ}
\]
Optimal Lead Time Padding

\[
TC[T_p] = \left( \sum_{t=0}^{T_p} r v Q T_p P[t] \right) - \left( \sum_{t=0}^{T_p} r v Q t P[t] \right) + \left( \sum_{t=T_p+1}^{\infty} C_d t P[t] \right) - \left( \sum_{t=T_p+1}^{\infty} C_d T_p P[t] \right)
\]

\[
\frac{dT C[T_p]}{dT_p} = r v Q \left( \sum_{t=0}^{T_p} P[t] \right) - (0) + (0) - \left( C_d \sum_{t=T_p+1}^{\infty} T_p P[t] \right) = 0
\]

\[
r v Q \left( Prob[NoStockout] \right) - \left( C_d \left( Prob[Stockout] \right) \right) = 0
\]

\[
r v Q \left( CSL^* \right) = C_d \left( 1 - CSL^* \right)
\]

\[
CSL^* = \frac{C_d}{C_d + r v Q}
\]
Optimal Lead Time Padding

Example:

\[ v = \$5.00/\text{unit} \quad Q = 1000 \text{ units} \]
\[ r = 36\% \text{ annual} \quad t' = 10 \text{ days} \]
\[ r_v = .005 \text{ dollars/unit/day} \quad \sigma = 3 \text{ days} \]
\[ C_d = \$500 \text{ per day} \quad (t \sim \text{normal}) \]

\[ \text{CSL}^* = \]
\[ k^* = \]
\[ T_p^* = \]
Benefits of MRP

- **Lower Inventory Levels**
  - Able to better manage components
  - Increased visibility

- **Fewer Stock outs**
  - Relationships are defined and explicit
  - Allows for coordination with MPS

- **Less Expediting**
  - Due to increased visibility

- **Fewer Production Disruptions**
  - Input needs are explicitly modeled
  - Plans are integrated
Shortcomings of MRP

- MRP is a scheduling, not a stockage, algorithm
  - Replaces the forecasting mechanism
  - Considers indentured structures

- MRP does not address how to determine lot size
  - Does not explicitly consider costs
  - Wide use of Lot for Lot in practice

- MRP systems do not inherently deal with uncertainty
  - User must enter these values – by item by production level
  - Typical use of "safety time" rather than "safety stock"

- MRP assumes constant, known leadtimes
  - By component and part and production level
  - But lead time is often a function of order size and other activity

- MRP does not provide incentives for improvement
  - Requires tremendous amount of data and effort to set up
  - Initial values are typically inflated to avoid start up issues
  - Little incentive to correct a system "that works"
MRP: Evolution of Concepts

**Simple MRP**
- Focus on "order launching"
- Used within production – not believed outside

**Closed Loop MRP**
- Focus on production scheduling
- Interacts with the MPS to create feasible plans

**MRP II [Manufacturing Resource Planning]**
- Focus on integrated financial planning
- Treats the MPS as a decision variable
- Capacity is considered (Capacity Resource Planning)

**Enterprise Resource Planning Systems**
- Common, centralized data for all areas
- Implementation is costly and effort intensive
- Forces business rules on companies
Questions? Comments? Suggestions?