Introduction to Stochastic Inventory Models and Supply Contracts

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Professor of Engineering Systems
Massachusetts Institute of Technology
Outline of the Presentation

◆ Introduction

◆ The Effect of Demand Uncertainty
  ◆ (s,S) Policy
  ◆ Supply Contracts
  ◆ Risk Pooling

◆ Practical Issues in Inventory Management

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Sources:
- Plants vendors
- Ports

Regional warehouses:
- Stocking points

Field warehouses:
- Stocking points

Customers, demand centers sinks

Supply

Production/purchase costs

Transportation costs

Inventory & warehousing costs

Image by MIT OpenCourseWare.
Goals:

Reduce Cost, Improve Service

- By effectively managing inventory:
  - Xerox eliminated $700 million inventory from its supply chain
  - Wal-Mart became the largest retail company utilizing efficient inventory management
  - GM has reduced parts inventory and transportation costs by 26% annually
Goals: Reduce Cost, Improve Service

- By not managing inventory successfully
  - In 1994, “IBM continues to struggle with shortages in their ThinkPad line” (WSJ, Oct 7, 1994)
  - In 1993, “Liz Claiborne said its unexpected earning decline is the consequence of higher than anticipated excess inventory” (WSJ, July 15, 1993)
  - In 1993, “Dell Computers predicts a loss; Stock plunges. Dell acknowledged that the company was sharply off in its forecast of demand, resulting in inventory write downs” (WSJ, August 1993)
Understanding Inventory

• The inventory policy is affected by:
  – Demand Characteristics
  – Lead Time
  – Number of Products
  – Objectives
    • Service level
    • Minimize costs
  – Cost Structure
Most companies treat the world as if it were predictable:

- Production and inventory planning are based on forecasts of demand made far in advance of the selling season.

- Companies are aware of demand uncertainty when they create a forecast, but they design their planning process as if the forecast truly represents reality.
Demand Forecast

• The three principles of all forecasting techniques:
  – Forecasting is always wrong
  – The longer the forecast horizon the worst is the forecast
  – Aggregate forecasts are more accurate
The Effect of Demand Uncertainty

• Most companies treat the world as if it were predictable:
  - Production and inventory planning are based on forecasts of demand made far in advance of the selling season
  - Companies are aware of demand uncertainty when they create a forecast, but they design their planning process as if the forecast truly represents reality

• Recent technological advances have increased the level of demand uncertainty:
  - Short product life cycles
  - Increasing product variety

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SnowTime Sporting Goods

- Fashion items have short life cycles, high variety of competitors
- SnowTime Sporting Goods
  - New designs are completed
  - One production opportunity
  - Based on past sales, knowledge of the industry, and economic conditions, the marketing department has a probabilistic forecast
  - The forecast averages about 13,000, but there is a chance that demand will be greater or less than this.
• Fashion items have short life cycles, high variety of competitors

• SnowTime Sporting Goods
  – New designs are completed
  – One production opportunity
  – Based on past sales, knowledge of the industry, and economic conditions, the marketing department has a probabilistic forecast
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SnowTime Demand Scenarios

![Demand Scenarios Graph]

- 8000: 10%
- 10000: 15%
- 12000: 20%
- 14000: 25%
- 16000: 15%
- 18000: 10%

Probability

Sales

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SnowTime Costs

• Production cost per unit (C): $80
• Selling price per unit (S): $125
• Salvage value per unit (V): $20
• Fixed production cost (F): $100,000
• Q is production quantity, D demand

• Profit = Revenue - Variable Cost - Fixed Cost + Salvage
SnowTime Best Solution

• Find order quantity that maximizes weighted average profit.

• Question: Will this quantity be less than, equal to, or greater than average demand?
What to Make?

- Question: Will this quantity be less than, equal to, or greater than average demand?
- Average demand is 13,100
- Look at marginal cost Vs. marginal profit
  - if extra jacket sold, profit is 125-80 = 45
  - if not sold, cost is 80-20 = 60
- So we will make less than average

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SnowTime Scenarios

• Scenario One:
  - Suppose you make 12,000 jackets and demand ends up being 13,000 jackets.
  - Profit = 125(12,000) - 80(12,000) - 100,000 = $440,000

• Scenario Two:
  - Suppose you make 12,000 jackets and demand ends up being 11,000 jackets.
  - Profit = 125(11,000) - 80(12,000) - 100,000 + 20(1000) = $335,000
SnowTime Expected Profit

Expected Profit

Profit

Order Quantity

$0

$100,000

$200,000

$300,000

$400,000

8000

12000

16000

20000

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SnowTime Expected Profit

Expected Profit

Profit

$0

$100,000

$200,000

$300,000

$400,000

Order Quantity

8000

12000

16000

20000

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• Tradeoff between ordering enough to meet demand and ordering too much
• Several quantities have the same average profit
• Average profit does not tell the whole story

• Question: 9000 and 16000 units lead to about the same average profit, so which do we prefer?
Probability of Outcomes

![Bar chart showing probability of outcomes for different cost values. The chart includes probability ranges at 0%, 20%, 40%, 60%, 80%, and 100%.

- For Q=9000, the probability distribution is shown.
- For Q=16000, the probability distribution is also shown.

Cost categories are labeled: -300000, -100000, 100000, 300000, 500000.

The chart illustrates the probability distribution for each cost category, with Q=9000 and Q=16000.

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Key Insights from this Model

• The optimal order quantity is not necessarily equal to average forecast demand
• The optimal quantity depends on the relationship between marginal profit and marginal cost
• As order quantity increases, average profit first increases and then decreases
• As production quantity increases, risk increases. In other words, the probability of large gains and of large losses increases

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Supply Contracts

Variable Production Cost = $100,000
Fixed Production Cost = $35

Wholesale Price = $80
Selling Price = $125
Salvage Price = $20

Manufacturer → Manufacturer DC → Retail DC → Stores

Image by MIT OpenCourseWare.

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Demand Scenarios

![Bar chart showing demand scenarios with probabilities for sales ranging from 8000 to 18000.]
Supply Contracts (cont.)

• Distributor optimal order quantity is 12,000 units
• Distributor expected profit is $470,000
• Manufacturer profit is $440,000
• Supply Chain Profit is $910,000

- IS there anything that the distributor and manufacturer can do to increase the profit of both?
Supply Contracts

Variable Production Cost = $100,000
Fixed Production Cost = $35

Wholesale Price = $80

Selling Price = $125
Salvage Price = $20

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Retailer Profit
(Buy Back=$55)
Retailer Profits
(Buy Back=$55)

$513,800

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Manufacturer Profit
(Buy Back=$55)
Manufacturer Profit
(Buy Back=$55)

$471,900

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Supply Contracts

Variable Production Cost = $100,000
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Salvage Price = $20

Image by MIT OpenCourseWare.

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Retailer Profit
(Wholesale Price $70, RS 15%)
Retailer Profit
(Wholesale Price $70, RS 15%)
Manufacturer Profit
(Wholesale Price $70, RS 15%)
Manufacturer Profit
(Wholesale Price $70, RS 15%)

$481,375
## Supply Contracts

<table>
<thead>
<tr>
<th>Strategy</th>
<th>Retailer</th>
<th>Manufacturer</th>
<th>Total</th>
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</thead>
<tbody>
<tr>
<td>Sequential Optimization</td>
<td>470,700</td>
<td>440,000</td>
<td>910,700</td>
</tr>
<tr>
<td>Buyback</td>
<td>513,800</td>
<td>471,900</td>
<td>985,700</td>
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<tr>
<td>Revenue Sharing</td>
<td>504,325</td>
<td>481,375</td>
<td>985,700</td>
</tr>
</tbody>
</table>

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Supply Contracts

Variable Production Cost = $100,000
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Wholesale Price = $80
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<td>481,375</td>
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<td>Global Optimization</td>
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</table>

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Supply Contracts: Key Insights

- Effective supply contracts allow supply chain partners to replace *sequential optimization* by *global optimization*.
- Buy Back and Revenue Sharing contracts achieve this objective through *risk sharing*.
- No one has an incentive to deviate from the contract terms.
Supply Contracts: Case Study

- Example: Demand for a movie newly released video cassette typically starts high and decreases rapidly
  - Peak demand last about 10 weeks
- Blockbuster purchases a copy from a studio for $65 and rent for $3
  - Hence, retailer must rent the tape at least 22 times before earning profit
- Retailers cannot justify purchasing enough to cover the peak demand
  - In 1998, 20% of surveyed customers reported that they could not rent the movie they wanted

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Supply Contracts: Case Study

• Starting in 1998 Blockbuster entered a revenue sharing agreement with the major studios
  - Studio charges $8 per copy
    Blockbuster pays 30-45% of its rental income
• Even if Blockbuster keeps only half of the rental income, the breakeven point is 6 rental per copy
• The impact of revenue sharing on Blockbuster was dramatic
  - Rentals increased by 75% in test markets
  - Market share increased from 25% to 31% (The 2nd largest retailer, Hollywood Entertainment Corp has 5% market share)
What are the drawbacks of RS?

- **Administrative Cost**
  - Lawsuit brought by three independent video retailers who complained that they had been excluded from receiving the benefits of revenue sharing was dismissed (June 2002)
  - The Walt Disney Company has sued Blockbuster accusing them of cheating its video unit of approximately $120 million under a four year revenue sharing agreement (January 2003)

- **Impact on sales effort**
  - Retailers have incentive to push products with higher profit margins
  - Automotive industry: automobile sales depends on retail effort
What are the drawbacks of RS?

- Retailer may carry substitute or complementary products from other suppliers
  - One supplier offers revenue sharing while the other does not
    - Substitute products: retail will push the product with high margin
    - Complementary products: retailer may discount the product offered under revenue sharing to motivate sales of the other product

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SnowTime Costs: Initial Inventory

- Production cost per unit (C): $80
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- Fixed production cost (F): $100,000
- Q is production quantity, D demand

Profit = Revenue - Variable Cost - Fixed Cost + Salvage
SnowTime Expected Profit

Expected Profit

Order Quantity

Profit

$0

$100,000

$200,000

$300,000

$400,000

8000

12000

16000

20000

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• Suppose that one of the jacket designs is a model produced last year.
• Some inventory is left from last year
• Assume the same demand pattern as before
• If only old inventory is sold, no setup cost

• **Question:** If there are 7000 units remaining, what should SnowTime do? What should they do if there are 10,000 remaining?
Initial Inventory and Profit

![Graph showing the relationship between production quantity and profit. The graph peaks at approximately 12,500 production quantity with a profit of around 500,000.]

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Initial Inventory and Profit

Profit

Production Quantity

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Initial Inventory and Profit

![Graph showing the relationship between production quantity and profit.](image-url)

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(s, S) Policies

- For some starting inventory levels, it is better to not start production
- If we start, we always produce to the same level
- Thus, we use an \((s, S)\) policy. If the inventory level is below \(s\), we produce up to \(S\).
- \(s\) is the reorder point, and \(S\) is the order-up-to level
- The difference between the two levels is driven by the fixed costs associated with ordering, transportation, or manufacturing
A Multi-Period Inventory Model

• Often, there are multiple reorder opportunities

• Consider a central distribution facility which orders from a manufacturer and delivers to retailers. The distributor periodically places orders to replenish its inventory
Case Study: Electronic Component Distributor

- Electronic Component Distributor
- Parent company HQ in Japan with world-wide manufacturing
- All products manufactured by parent company
- One central warehouse in U.S.
Case Study: The Supply Chain

Inbound

1) Order Processing
2) Forecasting
3) Order Processing
4) Production Planning

Outbound

1) Order Processing
2) Forecasting Replenishment

Order Flow
Product Flow

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Demand Variability: Example 1

Product Demand

<table>
<thead>
<tr>
<th>Month</th>
<th>Demand (000's)</th>
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<tbody>
<tr>
<td>Apr</td>
<td>150</td>
</tr>
<tr>
<td>May</td>
<td>75</td>
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<tr>
<td>Jun</td>
<td>225</td>
</tr>
<tr>
<td>Jul</td>
<td>100</td>
</tr>
<tr>
<td>Aug</td>
<td>150</td>
</tr>
<tr>
<td>Sep</td>
<td>50</td>
</tr>
<tr>
<td>Oct</td>
<td>125</td>
</tr>
<tr>
<td>Nov</td>
<td>61</td>
</tr>
<tr>
<td>Dec</td>
<td>48</td>
</tr>
<tr>
<td>Jan</td>
<td>53</td>
</tr>
<tr>
<td>Feb</td>
<td>104</td>
</tr>
<tr>
<td>Mar</td>
<td>45</td>
</tr>
</tbody>
</table>

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Demand Variability: Example 1

Histogram for Value of Orders Placed in a Week

- Frequency
- Value of Orders Placed in a Week

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Reminder:

The Normal Distribution

Average = 30

Standard Deviation = 5

Standard Deviation = 10

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The DC holds inventory to:

- Satisfy demand during lead time
- Protect against demand uncertainty
- Balance fixed costs and holding costs
The Multi-Period Inventory Model

- Normally distributed random demand
- Fixed order cost plus a cost proportional to amount ordered.
- Inventory cost is charged per item per unit time
- If an order arrives and there is no inventory, the order is lost
- The distributor has a required service level. This is expressed as the likelihood that the distributor will not stock out during lead time.
- Intuitively, how will this effect our policy?
A View of \((s, S)\) Policy

Inventory Position

Lead Time

Time

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(s, S) Policy: Whenever the inventory position drops below a certain level, \( s \), we order to raise the inventory position to level \( S \).

The reorder point is a function of:

- The Lead Time
- Average demand
- Demand variability
- Service level
Notation

- AVG = average daily demand
- STD = standard deviation of daily demand
- LT = replenishment lead time in days
- h = holding cost of one unit for one day
- SL = service level (for example, 95%). This implies that the probability of stocking out is 100%-SL (for example, 5%)
- Also, the **Inventory Position** at any time is the actual inventory plus items already ordered, but not yet delivered.
The reorder point has two components:
- To account for average demand during lead time:
  \[ LT \times AVG \]
- To account for deviations from average (we call this *safety stock*):
  \[ z \times STD \times \sqrt{LT} \]
where \( z \) is chosen from statistical tables to ensure that the probability of stockouts during leadtime is 100%-SL.
Example

- The distributor has historically observed weekly demand of:
  \[ \text{AVG} = 44.6 \quad \text{STD} = 32.1 \]
  Replenishment lead time is 2 weeks, and desired service level \( SL = 97\% \)

- Average demand during lead time is:
  \[ 44.6 \times 2 = 89.2 \]

- Safety Stock is:
  \[ 1.88 \times 32.1 \times \sqrt{2} = 85.3 \]

- Reorder point is thus 175, or about 3.9 weeks of supply at warehouse and in the pipeline

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Fixed Order Schedule

• Suppose the distributor places orders every month

• What policy should the distributor use?

• What about the fixed cost?
Base-Stock Policy

Units

Time

Target Inventory

Expected UB

Expected LB

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Consider these two systems:

1. Supplier → Warehouse One → Market One
2. Supplier → Warehouse Two → Market Two
3. Supplier → Warehouse → Market One → Market Two

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Risk Pooling

• For the same service level, which system will require more inventory? Why?
• For the same total inventory level, which system will have better service? Why?
• What are the factors that affect these answers?
Risk Pooling Example

• Compare the two systems:
  - two products
  - maintain 97% service level
  - $60 order cost
  - $.27 weekly holding cost
  - $1.05 transportation cost per unit in decentralized system, $1.10 in centralized system
  - 1 week lead time
## Risk Pooling Example

<table>
<thead>
<tr>
<th>Week</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
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<tbody>
<tr>
<td>Prod A, Market 1</td>
<td>33</td>
<td>45</td>
<td>37</td>
<td>38</td>
<td>55</td>
<td>30</td>
<td>18</td>
<td>58</td>
</tr>
<tr>
<td>Prod A, Market 2</td>
<td>46</td>
<td>35</td>
<td>41</td>
<td>40</td>
<td>26</td>
<td>48</td>
<td>18</td>
<td>55</td>
</tr>
<tr>
<td>Prod B, Market 1</td>
<td>0</td>
<td>2</td>
<td>3</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>3</td>
<td>0</td>
</tr>
<tr>
<td>Product B, Market 2</td>
<td>2</td>
<td>4</td>
<td>0</td>
<td>0</td>
<td>3</td>
<td>1</td>
<td>0</td>
<td>0</td>
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## Risk Pooling Example

<table>
<thead>
<tr>
<th>Warehouse</th>
<th>Product</th>
<th>AVG</th>
<th>STD</th>
<th>CV</th>
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<tbody>
<tr>
<td>Market 1</td>
<td>A</td>
<td>39.3</td>
<td>13.2</td>
<td>.34</td>
</tr>
<tr>
<td>Market 2</td>
<td>A</td>
<td>38.6</td>
<td>12.0</td>
<td>.31</td>
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<tr>
<td>Market 1</td>
<td>B</td>
<td>1.125</td>
<td>1.36</td>
<td>1.21</td>
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<tr>
<td>Market 2</td>
<td>B</td>
<td>1.25</td>
<td>1.58</td>
<td>1.26</td>
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## Risk Pooling Example

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<th>Product</th>
<th>AVG</th>
<th>STD</th>
<th>CV</th>
<th>s</th>
<th>S</th>
<th>Avg. Inven.</th>
<th>% Dec.</th>
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<tbody>
<tr>
<td>Market 1</td>
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<td>.34</td>
<td>65</td>
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<td>Market 2</td>
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<td>38.6</td>
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<tr>
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<td>B</td>
<td>1.25</td>
<td>1.58</td>
<td>1.26</td>
<td>5</td>
<td>27</td>
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<tr>
<td>Cent.</td>
<td>A</td>
<td>77.9</td>
<td>20.7</td>
<td>.27</td>
<td>118</td>
<td>226</td>
<td>132</td>
<td>26%</td>
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<tr>
<td>Cent</td>
<td>B</td>
<td>2.375</td>
<td>1.9</td>
<td>.81</td>
<td>6</td>
<td>37</td>
<td>20</td>
<td>33%</td>
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</table>
• Centralizing inventory control reduces both safety stock and average inventory level for the same service level.

• This works best for
  – High coefficient of variation, which reduces required safety stock.
  – Negatively correlated demand. Why?

• What other kinds of risk pooling will we see?
To Centralize or not to Centralize

• What is the effect on:
  – Safety stock?
  – Service level?
  – Overhead?
  – Lead time?
  – Transportation Costs?
Inventory Management: Best Practice

- Periodic inventory review policy (59%)
- Tight management of usage rates, lead times and safety stock (46%)
- ABC approach (37%)
- Reduced safety stock levels (34%)
- Shift more inventory, or inventory ownership, to suppliers (31%)
- Quantitative approaches (33%)

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Changes In Inventory Turnover

- Inventory turns increased by 30% from 1995 to 1998
- Inventory turns increased by 27% from 1998 to 2000
- Overall the increase is from 8.0 turns per year to over 13 per year over a five year period ending in year 2000.
## Inventory Turnover Ratio

<table>
<thead>
<tr>
<th>Industry</th>
<th>Upper Quartile</th>
<th>Median</th>
<th>Lower Quartile</th>
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<tbody>
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<td>Dairy Products</td>
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<td>19.3</td>
<td>9.2</td>
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<tr>
<td>Electronic Component</td>
<td>9.8</td>
<td>5.7</td>
<td>3.7</td>
</tr>
<tr>
<td>Electronic Computers</td>
<td>9.4</td>
<td>5.3</td>
<td>3.5</td>
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<tr>
<td>Books: publishing</td>
<td>9.8</td>
<td>2.4</td>
<td>1.3</td>
</tr>
<tr>
<td>Household audio &amp; video equipment</td>
<td>6.2</td>
<td>3.4</td>
<td>2.3</td>
</tr>
<tr>
<td>Household electrical appliances</td>
<td>8.0</td>
<td>5.0</td>
<td>3.8</td>
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<tr>
<td>Industrial chemical</td>
<td>10.3</td>
<td>6.6</td>
<td>4.4</td>
</tr>
</tbody>
</table>
Factors that Drive Reduction in Inventory

- Top management emphasis on inventory reduction (19%)
- Number of SKUs in the warehouse (10%)
- Improved forecasting (7%)
- Use of sophisticated inventory management software (6%)
- Coordination among supply chain members (6%)
- Others

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Factors that will Drive Inventory Turns Change by 2000

- Better software for inventory management (16.2%)
- Reduced lead time (15%)
- Improved forecasting (10.7%)
- Application of SCM principals (9.6%)
- More attention to inventory management (6.6%)
- Reduction in SKU (5.1%)
- Others