Random Networks and Percolation

- Percolation, cascades, pandemics
- Properties, Metrics of Random Networks
- Basic Theory of Random Networks and Cascades
- Watts Cascades
- Analytic Model of Watts Cascades
Types of Percolation Models

• “Short loop” models (Stauffer, Grimmett, Morris) usually assume regular network structure or same nodal degree for all nodes
• “Long loop/no loop” models (Newman, Calloway, Watts) usually assume random tree-like structure
• “Collective action” models (Schelling, Granovetter) assume $k = n$ (all nodes see all other nodes)
• “Local action” models assume $z \ll n$ (Watts, etc.)
• “Threshold models” (Morris, Schelling, Granovetter, Watts) assume that a node changes state when more than a threshold fraction of neighboring nodes have changed state
  – Threshold models are equivalent to deterministic two-person games (Lopez-Pintado) (Morris)
  – Disease spreading (SIR) assumes a threshold number of neighbors
• Mixed collective/local models have also been proposed (Valente) for diffusion of innovations
Percolation Contexts

- Spread of diseases (Watts and others) (local)
- Propagation of rumors (Newman, Watts, Calloway) (local) - scary talk about surprises
- Success of “blockbusting” (Schelling) (collective)
- Decision to join a riot (Granovetter) (collective)
- Adoption of innovations (Rogers, Valente) (both)
- In each case, nodes are assumed to be different in their susceptibility - an important issue for sociologists
- **Thresholds** are used to model these differences
Diffusion of Pandemic Diseases

- Model assumes disease starts from a point and travels in two modes: local commuting and international air travel
- Disease follows SIR or SIS model, with parameters that need to be estimated for each outbreak
- Procedure is to run the model with different trial parameters and see which ones best match time of outbreaks in different main airline destinations
- Model has been built up over about 10 years of IATA, census, and local transportation data
- Prediction that H1N1 would peak in US in October at low levels
- Prediction that it originated in Mexico, not a pig farm in MN, etc.
- http://cnets.indiana.edu/tag/epidemic-modeling
Diffusion of the Black Death: Slow

Image by MIT OpenCourseWare.

Ralph's World Civilizations, Chapter 13
SIR Model

Values of $R_0$ of well-known infectious diseases[1]

<table>
<thead>
<tr>
<th>Disease</th>
<th>Transmission</th>
<th>$R_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Measles</td>
<td>Airborne</td>
<td>12–18</td>
</tr>
<tr>
<td>Pertussis</td>
<td>Airborne droplet</td>
<td>12–17</td>
</tr>
<tr>
<td>Diphtheria</td>
<td>Saliva</td>
<td>6–7</td>
</tr>
<tr>
<td>Smallpox</td>
<td>Social contact</td>
<td>5–7</td>
</tr>
<tr>
<td>Polio</td>
<td>Fecal-oral route</td>
<td>5–7</td>
</tr>
<tr>
<td>Rubella</td>
<td>Airborne droplet</td>
<td>5–7</td>
</tr>
<tr>
<td>Mumps</td>
<td>Airborne droplet</td>
<td>4–7</td>
</tr>
<tr>
<td>HIV/AIDS</td>
<td>Sexual contact</td>
<td>2–5[2]</td>
</tr>
<tr>
<td>SARS</td>
<td>Airborne droplet</td>
<td>2–5[3]</td>
</tr>
<tr>
<td>Influenza</td>
<td>Airborne droplet</td>
<td>2–3[4]e</td>
</tr>
<tr>
<td>H1N1</td>
<td>Airborne droplet</td>
<td>1.5?</td>
</tr>
</tbody>
</table>

\[
\begin{align*}
\dot{S} &= -\beta IS \\
\dot{I} &= \beta IS - \nu I \\
\dot{R} &= \nu I \\
R_0 &= \frac{\beta}{\nu} "\text{Basic Reproduction Number}" 
\end{align*}
\]

http://en.wikipedia.org/wiki/Compartmental_models_in_epidemiology
http://en.wikipedia.org/wiki/Basic_reproduction_number
Transport Networks Cause Fast Diffusion

- US air travel
- US local commuting
- Global air travel
Using Data and Model to Find Parameters

![Graph showing the likelihood of epidemic parameter values with a color-coded 3D plot. The x-axis represents Ro values from 0.5 to 2.5, the y-axis represents IP values from 0 to 0.8, and the z-axis represents likelihood values from 0.0 to 0.4. The graph is color-coded with different shades indicating various likelihood ranges.](image-url)
Adoption of Innovations - Rogers

Basic idea: later adopters wait until more have adopted first. Gives rise to threshold models of diffusion and percolation.
Adoption of Hybrid Corn - Rogers

The number of new adopters each year, and the cumulative number of adopters, of hybrid seed corn in two Iowa communities.
What’s Interesting About Random Networks

• They represent one extreme of networks
  – Another is regular structures like grids or arrays
  – Another is “designed” networks with rational but not necessarily regular structure

• They can be analyzed mathematically (“light’s better”)
• The non-randomness of other networks can sometimes be measured by comparing metrics with random networks of similar size and density
• Some real networks are more random than one would imagine
• Some random networks harbor non-random properties
Basic Theory

- Network has $n$ nodes
- A pair of nodes is linked (both ways) with probability $p$
- The number of links in the network $m = pn(n-1)/2$
  - Some fraction of the number if all nodes were linked, counting each pair of nodes once
- The average nodal degree $z = 2m/n = pn$
- The clustering coefficient $C = pr(2$ neighbors linked$) = pr(any$ pair$ linked) = p = z/n$
- For given $z$, $C$ goes down as the size of the network grows
- For many properties “$P$” we find that $P$ “suddenly appears” when tracked according to some network parameter like $z$
  - “Sudden appearance” is usually called a phase transition
  - The most common example $P$ is connectedness
Subgraph Shapes in Finite Random Networks in Matlab

• When $z = 1$, about one third of the nodes are isolated and have $k = 0$ while another third have $k = 1$, implying lots of linked pairs of nodes
• Clusters, if any, have $z \sim 2$ and contain the last third
  – Small stars, chains, trees
  – Few closed loops
• To get a big connected cluster, we need $z > 1$ for the graph as a whole and $z > 2$ for a connected cluster because $z$ of a tree is $\sim 2$
  – For a tree, $m = n - 1$, $z = 2m/n \therefore z \approx 2$
Degree Distribution of ER Random Network

- For large $n$ the degree distribution is Poisson, and $z=np$ is the only adjustable parameter

$$p_k = \binom{n-1}{k} p^k (1-p)^{n-k-1} \approx e^{-z} \frac{z^k}{k!} \text{ if } n \to \infty$$

- This looks roughly Gaussian for large $z$, highly peaked for small $z$
- Standard deviation $\sigma = \sqrt{z}$

$z = 10$

- $p_0 = e^{-1} = 0.3679$
- $p_1 = e^{-1} = 0.3679$
- $p_2 = e^{-1}/2 = 0.1359$
- $p_3 = e^{-1}/6 = 0.0453$

$z = 1.006$
Average Path Length and Network Diameter

- Typical node has $z$ neighbors
- Each of them has $z$ neighbors (assumes somewhat tree-like structure, true when $z$ is not much bigger than 1)
- At distance $l$ there are $\sim z^l$ neighbors (if $z$ is small so that the network is mainly tree-like)
- If $d =$ the shortest distance all the way across a network of $n$ nodes, then $z^d \sim n$ and $d \sim \ln(n)/\ln(z)$
- Average path length < diameter so $l \sim \ln(n)/\ln(z)$
- Exact formula for APL: $< \ell > = \frac{\ln(n)-\gamma}{\ln(z)} + 0.5$
  
\[ \gamma = \text{Euler's number} = 0.5771 \]
Percolation and Cascades

• Both terms are ~synonymous with emergence of a “giant cluster”
  – In an infinitely large random network, the size of a connected cluster is a non-zero % of the total number of nodes
  – In a finite network, the cluster size is comparable to the size of the network
• Giant clusters appear if the network is dense enough
• The proven threshold for E-R is $z = 1$
Percolation, Cascades, Rumors

- A network consists of nodes that can be “flipped” from their initial state (off) to another state (on) depending on their “vunerability”
- A “seed” node (or in some models, a set of seed nodes) is arbitrarily switched from off to on
- Subsequently, other neighboring nodes may flip, depending on model assumptions
- The cascade will not permeate the whole network unless the network “percolates” or is connected with probability = 1
- Even if it is connected, it still may not percolate
Vulnerability and Stability

- A node is "vulnerable" if one flipped neighbor can flip it $k \leq K^*$
- A stable node is "first order stable" if two flipped neighbors can flip it $K^* < k \leq 2K^*$
- A stable node is "second order stable" if three flipped neighbors can flip it $2K^* < k \leq 3K^*$
- etc
Percolation Theory for Sparse Random Graphs

- Derived by Newman and others using generating functions
- Recreates and extends the Molloy-Reed criterion
- Extended by Watts
- Assumes graphs (or vulnerable subgraphs) are trees and networks are of infinite size

All nodes vulnerable

\[ \sum_{k=0}^{\infty} k(k-1)p_k = z \quad \text{Molloy-Reed criterion} \]

Vuln with \( pr = b \)

\[ b \sum_{k=0}^{\infty} k(k-1)p_k = z \]

Vuln is fct of \( k \)

\[ \sum_{k=0}^{\infty} k(k-1)\rho_k p_k = z \]

Watts rumor cascade model:

\[ \rho_k = \begin{cases} 
1 \text{ for } k \leq K^* \\
0 \text{ for } k > K^* 
\end{cases} \]
Example Percolation on a Tree

\[ z_i = \text{excess degrees at step } i \]
\[ n_i = \text{ith neighbors} \]

\[ z_1 = 3 \quad n_1 = 3 \]
\[ z_2 = 2 \quad n_2 = z_2 \times n_1 = 2 \times 3 = 6 \]
\[ z_3 = 2 \quad n_3 = z_3 \times n_2 = z_3 \times z_2 \times n_1 = 2 \times 2 \times 3 = 12 \]

For E - R, avg excess degree of a neighbor = \( z - 1 \)
Single Node Seed, No Threshold

Number of nodes hit on first step =
number of edges out from seed = $S_z$
Multi-Node Seed, Threshold

Number of nodes hit < number of edges out because some nodes are hit multiple times, allowing stable nodes to be flipped
Two Steps, Multi-Node Seed, Threshold

1 ≤ k ≤ K*: flip if ≥ 1 neighbor flips

K* + 1 ≤ k ≤ 2K*: flip if ≥ 2 neighbors flip

Seed

First Step

Second Step

K* = 4
Watts’ Cascade Diagram

Theoretical boundary: infinite nodes

Simulation boundary: 10000 nodes

No Global Cascades

Network is too densely connected

Global Cascades

Network is disconnected

Lower threshold: more likely to cascade

\[ K^* = \left\lfloor \frac{1}{\phi} \right\rfloor \]

Watts, PNAS April 2002
Vulnerable Clusters in Finite E-R Networks

Network is one big stable cluster with very few isolated vulnerable nodes

Network is ~80% stable nodes in one cluster plus ~19% isolated vulnerable nodes and 1% small vulnerable clusters

Network is ~100% vulnerable nodes, isolated or in small clusters
Watts Theory and Simulations

- **Theory**
  - Global cascades region: “small” seed can start a global cascade
  - No global cascades region: “small” seed cannot start a global cascade

- **Simulations on network with 10000 nodes**
  - Global cascades region: seed of one node can start a global cascade
  - No global cascades region: seed of one node cannot start a global cascade
Cascades in Finite E-R Networks Can Happen in the No Global Cascades Region

Simulations: Necessary Seed Sizes ($n = 4500$)
Simulations: Threshold Seed Size - A Phase Transition

Likelihood of TNC vs Size of Seed

$n = 4500$, $z = 11.5$, $K^* = 4$

“Threshold” seed size = 215
Typical Cascade Trajectories Throughout Transition Range of Seed Size

\[ n = 4500, \ z = 11.5, \ K^* = 4 \]

“Near death” phenomenon

\[ S = 200 \ (\text{no TNCs}) \]
\[ S = 210 \ (\text{no TNCs}) \]
\[ S = 220 \ (30\% \ TNCs) \]
\[ S = 220 \ (70\% \ no \ TNCs) \]
\[ S = 230 \ (90\% \ TNCs) \]
\[ S = 230 \ (90\% \ TNCs) \]
\[ S = 240 \ (100\% \ TNCs) \]
\[ S = 270 \ (100\% \ TNCs) \]
\[ S = 300 \ (100\% \ TNCs) \]
Theory

\[ p_k = \sum_{i=0}^{k} p_S(i,S)p_{nS}(k-i,n-S) \]

\[ p_k = \sum_{i=0}^{k} \binom{S}{i} p^i (1-p)^{s-i} \binom{N-1-S}{k-i} p^{k-i} (1-p)^{N-1-S(k-i)} \]

Any unflipped node: n-S of them

Any unflipped node: n-S-F1 of them

First flipped set = F1

Seed = S

Network = n
Theory Step 2

\[ p_k = \sum_{i=0}^{k} \sum_{i'=0}^{k-i} \binom{S}{i} p^i (1-p)^{S-i} \binom{F_1}{i'} p_{F_1}^{i'} (1-p_{F_1})^{F_1-i'} \]

\[ \times \binom{n-S-F_1-1}{k-i-i'} p_{nSF_1}^{k-i-i'} (1-p_{nSF_1})^{n-S-F_1-1-(k-i-i')} \]

\[ p_{F_1} = z_{F_1} / n \text{ reflects available edges from } F_1 \]

\[ p_{nSF_1} \text{ reflects larger } p \text{ of unflipped nodes} \]
Theory: Cascades in “Global Cascades”
Region - Seed = 1 Node

Number Flipped vs $z$ for $S = 1$, $n = 4500$

- $K^* = 4$
- $K^* = 5$
- $K^* = 6$
- $K^* = 7$
- $K^* = 8$
Theoretical and Simulation Results: Cascade in “Global Cascades” Region

- Theory and Simulation (avg of 10 runs)

- n = 4500, z = 11.5, S = 1, K* = 9

- Theory and Simulation (avg of 10 runs)

- Step

- Flip by Step (T)
- Flip Total (T)
- Flip by Step (S)
- Flip Total (S)

T = theory
S = simulation
Theory and Simulations: A Cascade in “No Global Cascades” Region

Theory and Simulations
\[ n = 4500, z = 11.5, K^* = 4, S = 250 \]

![Graph showing data comparison between cumflipped, cumflipped(T), numflippedthisstep(S), and numflippedthisstep(T).]
Theory: Ability to Predict Threshold Seed Size

S Transition: Theory and Simulations
n = 4500, z = 11.5, K* = 5

S Transition: Theory and Simulations
n = 4500, z = 14.56, K* = 5

S Transition: Theory and Simulations
n = 4500, z = 14.56, K* = 4

S Transition: Theory and Simulations
n = 4500, z = 11.5, K* = 4
At $S = 215$, Failure Most of the Time
Occasionally, Success: Why?

Success
\[ n = 4500, z = 11.5, k^* = 4, S = 215 \]

"Near death"
Cause of Wake-up After Near Death

- Caused by a critical mass phenomenon
- Near death only a few nodes flip on each step
- At most they can hit one node each since, for so few nodes, the likelihood of multiple hits is about zero
- So only nodes that are one hit short of flipping have any chance to flip during this phase
- This chance is proportional to how many one-shorts there are on any step and how many net edges $F_j$ has
- This population is growing but at the same time the number of flippers is falling
Derivation of Critical Mass

\[ pr(\text{a node in the network links to } F_j) = \frac{F_j z_{F_j}}{n} \]

\# nodes with edges to \( F_j \) = \( F_j z_{F_j} \)

= \# nodes that will be hit by \( F_j \)

fraction of these that will flip = 

fractional representation of one short in the network = \( \frac{N_{OS}}{n} \)

number of nodes that \( F_j \) flipped nodes will flip = \( \frac{N_{OS} F_j z_{F_j}}{n} \)

If each node in \( F_j \) flips one node, the cascade is self-sustaining.

So \( 1 = \frac{N_{OS} z_{F_j}}{n} \)

or \( N_{OS} = n / z_{F_j} \)

or \( F_j \cdot N_{OS} = F_j \cdot n / z_{F_j} \)
Theory and Simulations: Evolution of max One Short Failures (avg of 20 runs)

If one short exceeds the bound, a TNC almost always occurs.
If one short does not exceed the bound, a TNC almost never occurs.
Variation in one short can cause a TNC when mean is below bound.
When $r > 0$ Cascades Occur for Bigger $z$

Increasing $r$ generates larger vulnerable clusters
References

More References


• Lopez-Pintado, D., “Diffusion in Complex Social Networks,” *Games and Economic Behavior*, forthcoming


• Schelling, T.C., “A Study of Binary Choices with Externalities,” *J. Conflict Resolution*, 17 (3) 381- 428 (1973)

More References

- http://cnets.indiana.edu/tag/epidemic-modeling
Backups
Generalized Random Networks

- E-R random network has a Poisson degree distribution
- Random networks can be built with arbitrary degree distributions, but software is required
- Newman says that it is better to generate a specific degree sequence from the distribution and then generate a network with that degree sequence in order to guarantee that the software uses the same degree sequence all the way through the generating process
Subgraph Shapes vs $p$

$p \sim n^a, z \sim n^{a+1}$

<table>
<thead>
<tr>
<th>$z$</th>
<th>0</th>
<th>$n^{-1}$</th>
<th>$n^{-1/2}$</th>
<th>$n^{-1/3}$</th>
<th>$n^{-1/4}$</th>
<th>1</th>
<th>$n^{1/3}$</th>
<th>$n^{1/2}$</th>
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<tbody>
<tr>
<td>$p$</td>
<td>0</td>
<td>$n^{-2}$</td>
<td>$n^{-3/2}$</td>
<td>$n^{-4/3}$</td>
<td>$n^{-5/4}$</td>
<td>$n^{-1}$</td>
<td>$n^{-2/3}$</td>
<td>$n^{-1/2}$</td>
</tr>
</tbody>
</table>

$a$

$-\infty, -2, -\frac{3}{2}, -\frac{4}{3}, -\frac{5}{4}, -1, -\frac{2}{3}, -\frac{1}{2}$

$z$ for $n = 1000$: 0, 0.001, 0.0316, 0.1, 0.178, 1, 10, 31.6

The threshold probabilities at which different subgraphs appear in a random graph. For $pn^{3/2} \to 0$ the graph consists of isolated nodes. For $p \sim n^{-3/2}$ trees of order 3 appear, while for $p \sim n^{-4/3}$ trees of order 4 appear, but not many. At $p \sim n^{-1}$ trees of all orders are present and at the same time cycles of all orders appear, but again, not many. The probability $p \sim n^{-2/3}$ marks the appearance of complete subgraphs of order 4 and $p \sim n^{-1/2}$ corresponds to complete subgraphs or order 5. As $a$ approaches 0 the graph contains complete subgraphs of increasing order.
Site and Bond Percolation on Regular Graphs

• “The most common percolation model is to take a regular lattice, like a square lattice, and make it into a random network by randomly ‘occupying’ sites (vertices) or bonds (edges) with a statistically independent probability $p$. At a critical threshold $p_c$, long-range connectivity first appears, and this is called the percolation threshold.” [see wikipedia reference “percolation threshold”]

• For a square grid, $p_c = 0.5$ for bond percolation and $p_c = 0.59274621$ for site percolation
Percolation Step by Step

Regular Networks: Occupancy

Each node has 2 neighbors.
It must be linked to both for there to be a chance of a giant cluster. So $p_c = 1$.

Each node has 4 neighbors.
It must be linked to at least 2 for there to be a chance of a giant cluster. So $p_c = 0.5$.

Random Networks: $z$

In a random network with $n$ nodes each node has $n$ neighbors.
It must be linked to at least one for there to be a chance of a giant cluster. So $p_c = 1/n$. But $z = pn$ so this is the same as $z_c = 1$.

See Albert and Barabasi “Stat Mech of Complex Networks” for a detailed derivation.

There is no proof or formula for $p_c$ when $d > 2$ except for $d > 19$ and some special cases.

Scheduled for publication in 2007. for a detailed discussion
http://www.math.ubc.ca/~slade/
Vulnerable Clusters Multiply the Seed’s Search Efficiency and Effectiveness

By itself, a single seed cannot flip a stable node.

Vulnerable nodes have a few links to each other (average ~ 1.5) and more links to stable nodes outside their cluster. Working together, vulnerable nodes can flip stable nodes but most likely this happens only when vulnerable nodes co-exist in clusters.
Cluster-Hopping Creates TNCs When Vulnerable Clusters Are Small

Whitney, ICCS, 2007
Do Random Networks Have Cycles?

\[
\text{Expected } k - \text{cycles} = \frac{{n \choose k}}{2k} p^k \approx \frac{z^k}{2k} \text{ for } n \gg k
\]

Diestel, R., Graph Theory, 3rd edition online at <http://www.math.uni-hamburg.de/home/diestel/books/graph.theory/index.html> page 298
Seed is 2365

Loop Length | Number of Loops
---|---
3 | 261
4 | 2274
5 | 21114
6 | 204118
7 | 2028956
8 | 20581273

$n = 3000$
$z = 11.62$

Biggest cluster = 13

and cascades
Percolation Theory for Random Graphs

- Most theory assumes we are dealing with a sparse network that has few or no closed loops, especially no small closed loops.
- This is measured by the clustering coefficient, which is small for big random networks where the theory has been developed.
- If there is no clustering or small closed loops then it is easy to calculate how many neighbors, second neighbors, third neighbors, etc, a given node has because no node is its own third neighbor and the probability that a node is its own $n^{th}$ neighbor goes down as $n$ goes up.
- If there are more $n^{th}$ neighbors than $(n-1)^{th}$ neighbors for all $n$ and the network is tree-like, then there is a giant cluster.
- The calculations can be done for any random graph whose degree distribution is known, not just E-R random graphs, as long as there is negligible clustering.
Variants of the Theory

1. The percolation (or cascade) proceeds when a link is established between two nodes. This is basic simple percolation described on the previous slide.

2. The percolation proceeds if a link is established with a node that is “vulnerable”
   - A) Vulnerability can be a function of $k$ or it can be the same for all nodes (some number $0 \leq b \leq 1$)
   - B) Simple percolation has $b = 1$ for all nodes
   - C) Watts rumors cascade model has

\[
\begin{align*}
    b &= 1 \text{ for } k \leq K^* \\
    b &= 0 \text{ for } k > K^*
\end{align*}
\]
Derivation of Cascade Conditions (Newman)

Pick an edge leading from a node and follow it to a neighboring node.

What is the (excess) degree distribution of this neighbor?
If its degree = \( k \), its edges are \( k \)-times more numerous than if its degree = 1 (think of the edge list)
But the fraction of nodes with degree \( k \) is \( p_k \).
So the likelihood of encountering a node of degree \( k \) by this process is proportional to \( kp_k \)

Distribution of excess degrees of neighbor = \( q_{k-1} = \frac{kp_k}{\sum_{k=0}^{\infty} kp_k} \) or \( q_k = \frac{(k+1)p_{k+1}}{z} \)

\( \langle q \rangle = \text{avg excess degree of neighbor} = \sum_{k=0}^{\infty} kq_k = \frac{\sum_{k=0}^{\infty} (k+1)p_{k+1}}{z} = \frac{\sum_{k=0}^{\infty} k(k-1)p_k}{z} = \frac{< k^2 > - z}{z} = \frac{z_2}{z_1} \)
avg number of 2nd neighbors per 1st neighbor = \( z_2 = \langle k^2 \rangle - z \)

avg number of 3rd neighbors per 2nd neighbor = \( z_3 = z_2 = \langle k^2 \rangle - z \)

Avg number of \( m \)th neighbors = \( z_1 \left[ \frac{z_2}{z_1} \right]^{m-1} \)

This diverges when \( \frac{z_2}{z_1} = 1 \)

\[
\sum_{k=0}^{\infty} k(k-1)p_k \left( \frac{z_2}{z_1} \right)^{m-1} = 1
\]

\[
\sum_{k=0}^{\infty} k(k-1)p_k = z
\]

For E - R \( \langle k^2 \rangle = \langle k \rangle^2 = z^2 \)

So, for E - R this is the same as \( z = 1 \)

(See notes)
continuing

For the case where all nodes have vulnerability = b:

\[ b \sum_{k=0}^{\infty} k(k-1)p_k = z \]  

See notes

For the case where vulnerability is a function \( \rho_k \) of \( k \)

\[ \sum_{k=0}^{\infty} k(k-1)\rho_k p_k = z \]  

Watts rumor cascade model:

\[ \rho_k = \begin{cases} 
1 & \text{for } k \leq K^* \\
0 & \text{for } k > K^* 
\end{cases} \]

Supporting derivations of these typically use generating functions
Rules for Simulating Cascades

• Build a random network with some value of $z$ and set the value of $K^*$
• Choose a node at random (the seed) and flip it
• Find its neighbors and flip all that are vulnerable
• Find their neighbors and flip all that can be flipped
  – "Vulnerable" ones flip if one neighbor flipped
  – "First-order" stable ones flip if two neighbors flipped, etc
• Keep going until all nodes have flipped that can
• Use some criterion to say if a global cascade has happened or not
• Watts made a new network each time but I reused the network to save time. This permitted me to examine it.