Problems Day 10, F 2/16/2024

Topic 5: Homogeneous, linear, constant coefficient DEs (day 2)

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Problem 1. Consider the damped harmonic oscillator $2x'' + 8x' + 6x = 0$.

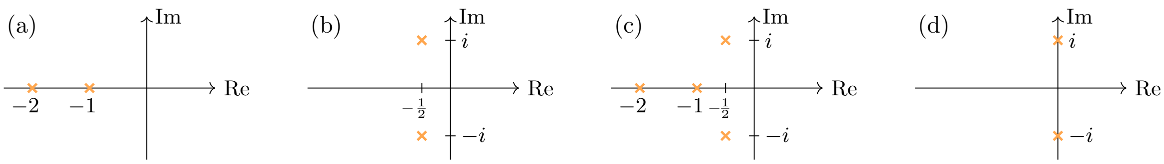
(a) What is the natural frequency of this oscillator?

(b) What type of damping (over, under or critical) does it have?

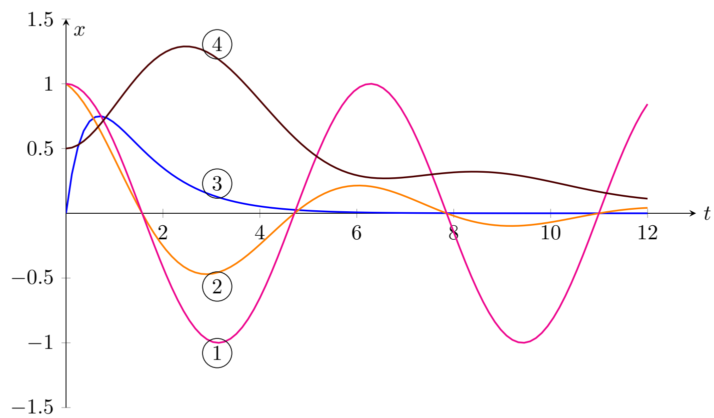
(c) How fast does it decay to 0?

Problem 2. Solve the DE $2x'' + 8x' + 6x = 0$ with initial conditions $x(0) = 1$, $x'(0) = 0$. (Hint: this is the same DE as in Problem 1.)

Problem 3. Each of the following pole diagrams show the characteristic roots of a linear, constant coefficient, homogeneous DE.



For each: Give the order of the system. Is it oscillatory? What is the exponential decay rate? Match the graphs below to the system.



Problem 4. Consider $z_1(t) = e^{(-2+3i)t}$, $z_2(t) = e^{(-2-3i)t}$.

(a) Why are z_1, z_2 called complex-valued functions?

(b) Show that the (very special) linear combinations

$$x_1 = \frac{1}{2} z_1 + \frac{1}{2} z_2, \quad x_2 = \frac{1}{2i} z_1 - \frac{1}{2i} z_2$$

are real valued.

Problem 5. Suppose x_1, x_2 are solutions to $x'' + 8x' + 7x = 0$. Verify the superposition principle for linear, homogeneous DEs. That is, show that $x = c_1 x_1 + c_2 x_2$ is also a solution for all constants c_1, c_2.