

## Problems Day 16, T 2/27/2024

Topic 8: Stability

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**Problem 1.** Say whether or not the systems  $P(D)x = f$  with the given roots are stable.

(a) Roots:  $-1, -1, -2, -3$

(b) Roots:  $-1, -1 \pm i, -2 \pm 3i$

(c) Roots:  $0, -1, -2$

(d) Roots:  $-1, -2, -3, 1 \pm i$

**Problem 2.** Are the following systems stable?

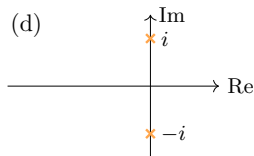
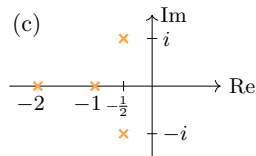
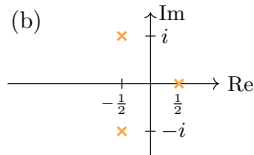
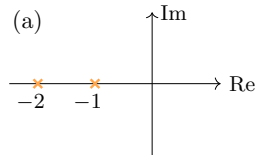
(a)  $3x'' + 4x' + 5x = f(t)$

(b)  $x' + 2x = f(t)$

(c)  $(D + 3I)(D - 3I)(D + 4I)x = f(t)$

(d)  $x''' + 2x'' + 3x' + 7x = f(t)$

**Problem 3.** For a system  $P(D)x = f$ , the pole diagram shows the roots in the complex plane. Do the following pole diagrams come from stable systems?



**Problem 4.** Suppose  $x = 8e^{2t}$  is a solution to  $P(D)x = 5e^{2t}$ ,  $x(0) = 8$ .

(a) Give a solution to  $P(D)x = 5e^{2(t-3)}$ ,  $x(3) = 8$ .

(b) Give a solution to  $P(D)x = 10e^{2(t-3)}$ ,  $x(3) = 16$ .

(c) If  $x_h(t) = c_1e^{-t} + c_2e^{-2t}$  solves  $P(D)x_h = 0$ , give a solution to  $P(D)x = 0$ ,  $x(3) = 1$ ,  $x'(3) = -1$ .

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ES.1803 Differential Equations

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