Problems Day 19, F 3/1/2024

Topic 9: Engineering language: input, gain, phase lag, frequency response (day 2 of 3)

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Continue from Topic 9, Day 1.

Problem 1. Consider the system $x'' + x' + 4x = 4\cos(5t)$.

(a) Find the sinusoidal response.

(b) Assume $\cos(5t)$ is the input. What are the gain and phase lag?

(c) Assume $4\cos(5t)$ is the input. What are the gain and phase lag?

Problem 2. A system is modeled by $x' + kx = kf(t)$, $k > 0$. We consider $f(t)$ to be the input.

(a) Solve the DE with input $B\cos(\omega t)$.

(b) What are the gain and phase lag for the system?

(c) Graph the gain. (Be sure to label your axes.)

Problem 3. A system modeled by a constant coefficient, linear DE has gain and phase lag as shown.
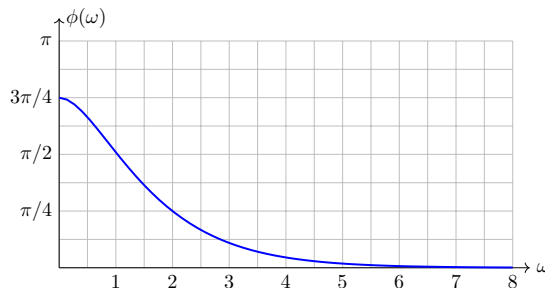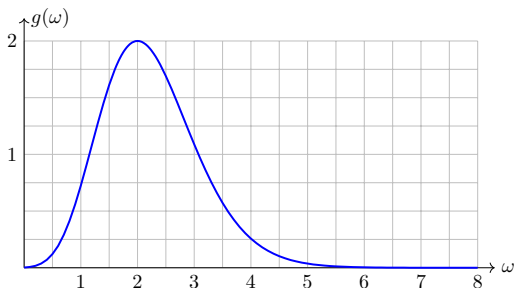
(a) If the input is $B\cos(2t)$, what is the periodic repsonse?

(b) If the input is $3\cos(2t) + 3\cos(6t) + 3\cos(8t)$, give a good approximation to the response.

(c) What input frequency has the biggest response?

Problem 4. Find all the resonant frequencies of the following systems.

(a) $x'' + x' + 9x = f(t)$, $f(t) =$ input.

(b) $x'' + 8x' + 7x = f(t)$, $f(t) =$ input.

(c) $x'' + 8x' + 7x = f'(t)$, $f(t) =$ input.

Problem 5. Consider the system $2x'' + 8x = f'(t)$, where $f(t)$ is considered the input.

(a) Find the periodic response to $f(t) = B\cos(\omega t)$, for all ω.

(b) Give formulas for the gain and phase lag.

(c) Plot the gain.

(d) Plot the response when $\omega = 2$. Why do we say $g(2) = \infty$.

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