Problems Day 2, T 2/6/2024
Topic 1: Introduction to differential equations (day 2)
Jeremy Orloff

Problem 1. Solve $\dfrac{dy}{dt} = ay$, a a constant.

Problem 2. Check that $y(t) = c_1 \cos(2t) + c_2 \sin(2t)$ (c_1, c_2 constants) solves $y'' + 4y = 0$.

What physical system does this model? (There are many possible answers.)

Problem 3. Interpret Newton's law of cooling in words: $T' = -k(T - E)$.

Problem 4. Solve $\dfrac{dy}{dt} = y^2$ with initial value $y(0) = 1$.

Graph the solution.

Problem 5. A curve $y = y(x)$ has the property that every tangent line goes through the point $(1, 0)$. Find a DE for this curve. Solve the DE to find all curves with this property.

Problem 6. Suppose Oryx (African antelope) have a natural growth rate of $k = 0.02$/year (made up number). Suppose they are "harvested" at a rate of $h = 1000$ oryx/year.

Model the population $x(t)$ by finding a DE from first principles using Δx and Δt.

How does your model change if $h = 10000 \sin(2\pi t)$?

What is happening if $h < 0$?

Problem 7. Interpret Hooke's law, $m\dfrac{d^2 x}{dt^2} = -kx$ in words.

What is the dimension of k?

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ES.1803 Differential Equations

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