Solutions Day 23, R 3/7/2024

Topic 10: Graphical methods: Direction fields, integral curves

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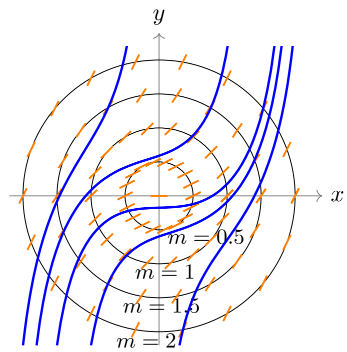
Problem 1. *Use isoclines to sketch a direction field for $y' = x^2 + y^2$. Add some integral curves*

Solution: Isoclines: $x^2 + y^2 = m$ = circle of radius $\sqrt{m}$. We show isoclines for $m =$ 0, 1, 1/2, 2. Then we add some integral curves.



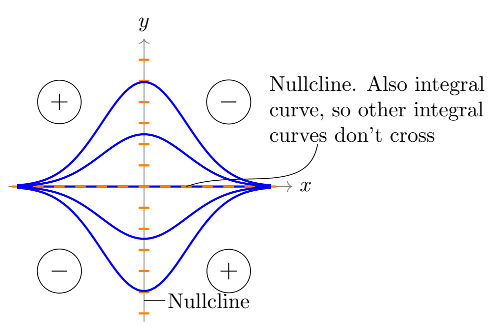
Problem 2. *For the DE, $y' = -xy$:*

Draw the nullclines. The function $y(x) \equiv 0$ is clearly a solution. Discuss how this relates to the nullclines.

The nullclines divide the xy-plane into 4 regions. Put a big ⊕ or ⊖ in each region to indicate whether the slope of integral curves is positive or negative.

Sketch in some integral curves.

Solution: Nullcline: $y' = -xy = 0 \quad \Rightarrow x = 0$ or $y = 0$, i.e., the axes. Since the slope field along the x-axis is tangent to the axis, it's an integral curve, i.e., the curve $y(x) \equiv 0$ has $y' = 0 = -xy$.



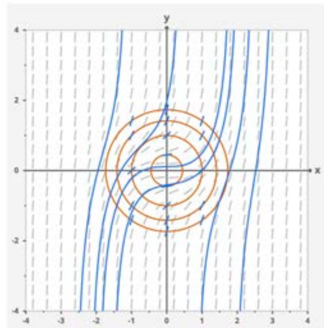
Problem 3. *Open the Isoclines mathlet:* `https://www.mathlets.org/mathlets/isoclines/`

Choose the DE $y' = y^2 - ax^2$.

(a) *Set $a = -1$. Use the m-slider to draw some isoclines. Click in the graph window to draw some integral curves.*

–Notice how the slope elements on an isocline are all alike.
–Notice how the integral curves follow the slope field.
–Compare the picture to your answer in Problem 1.

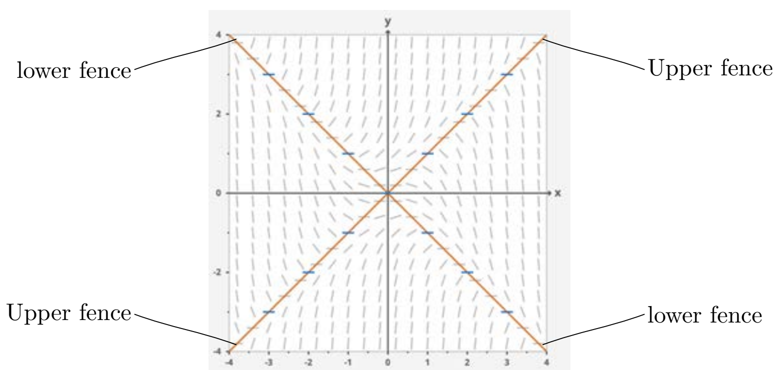
Solution:



(b) *Clear the graph window and set $a = 1$.*

Use the m-slider to draw the nullcline.
Which portions of the nullcline are upper fences? Lower fences?

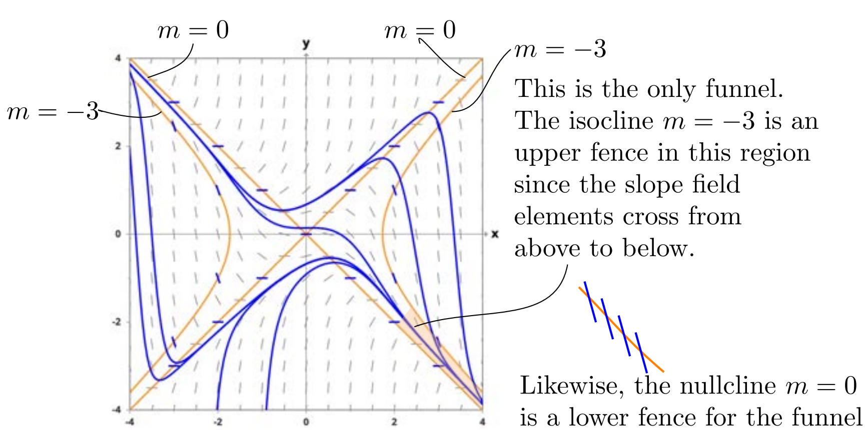
Solution:



The pieces of the nullcline in each quadrant form fences

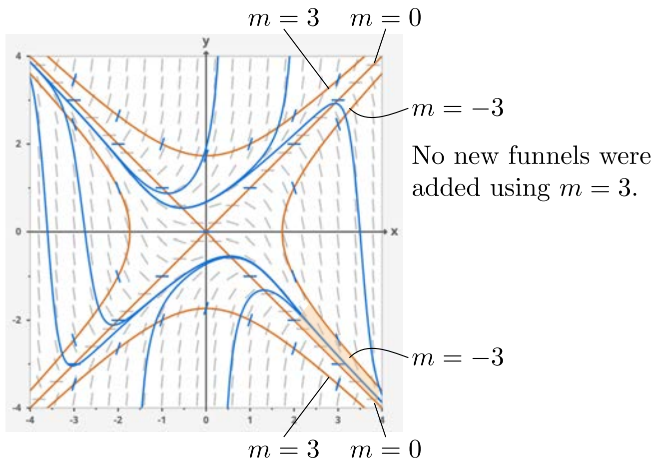
Add the isocline with $m = -3$. Identify any funnels. Check your answer by adding some solution curves.

Solution:



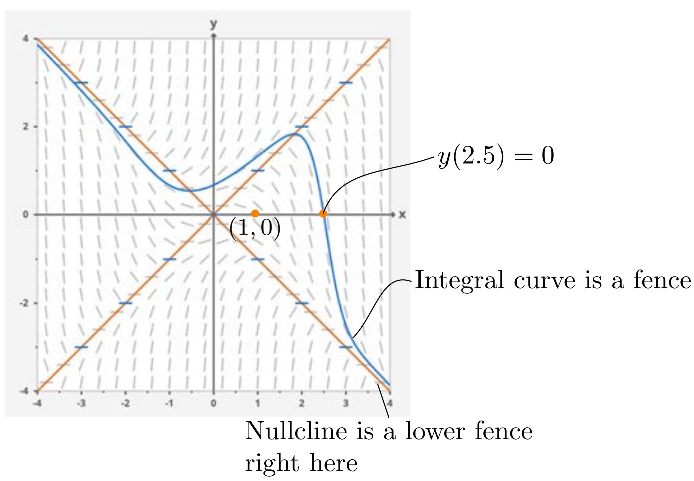
(c) *Add the isocline $m = 3$. Identify any new funnels. Check your answer by adding some solution curves.*

Solution:



(d) *Still with $a = 1$. Clear the graph. Draw the nullcline and the solution with $y(2.5) = 0$. Without drawing it, for the solution with $y(1) = 0$, estimate the value of $y(4)$. Check your answer by drawing the solution.*

Solution: The integral curve shown and the nullcline form a funnel in the 4th quadrant.



Since (1,0) is inside the funnel, the integral curve with $y(1) = 0$ must stay in the funnel. Therefore, $y(4) \approx -4$. (We can even say that $y(4)$ is slightly larger than -4.)