Day 25, M 3/11/2024

Topic 12: Autonomous systems (day 1 of 2)

Jeremy Orloff

1 Agenda

- Autonomous DEs: $y' = f(y)$
  - Understand behavior without solving exactly
  - Isoclines are horizontal lines
  - Time invariant
  - Equilibria where $f(y) = 0$, called critical points
  - Nullclines = integral curves = equilibrium solutions
  - Can summarize with the phase line
  - Bifurcation diagrams (tomorrow)

2 Autonomous DEs

$y' = f(y)$ is called autonomous.

Autonomous = self regulating, i.e., y controls how y changes.

3 Theory

- The equation $\dfrac{dy}{dt} = f(y)$ is separable: $\dfrac{dy}{f(y)} = dt$
- We have lost solutions where $f(y) = 0$. They are now called equilibrium solutions
- Values of y where $f(y) = 0$ are called critical points
- Can ask if each equilibrium is stable or unstable

Example 1. In this example, we will get to the phase line via the direction field.

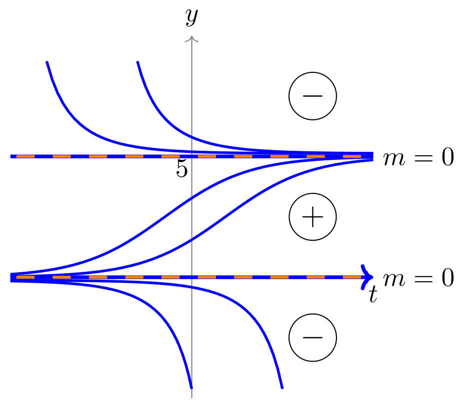
Consider $y' = 3(5 - y)y$ (logistic population model).

Plot the nullclines and add $\oplus$, $\ominus$ in regions where y' is positive or negative.

Draw some integral curves.

Draw the phase line.

Solution: Nullcline: $y' = 3(5 - y)y = 0 \longrightarrow y = 0$, $y = 5$, i.e., horizontal lines.



In this special case, nullclines are also integral curves (equilbrium solutions). It's easy to check that $y(t) \equiv 5$ is a solution.

Note: The other integral curves can't cross the horizontal integral curves $y = 5$ and $y = 0$.

To get the phase line, we reduce the above picture to just the y-axis.