Solutions Day 26, T 3/12/2024
Topic 12: Autonomous DEs (day 2)
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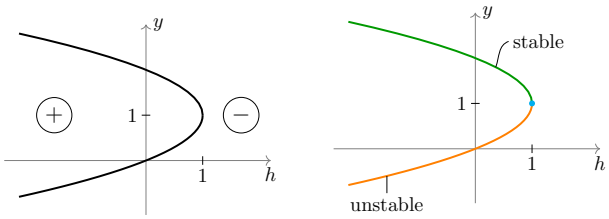
Problem 1. *Consider $y' = y(2-y) - h$, (h a constant).*

(a) *Draw the bifurcation diagram. (y vs. h)*

Solution: Critical points $y' = 0 \Leftrightarrow h = y(2-y) =$ sideways parabola.



(b) *I'm growing alfalfa and the amount $y(t)$ is modeled by the equation in Part (a). Here h is the constant rate that I harvest the alfalfa to feed to my horses. If I harvest at too great a rate, the alfalfa population will plummet and my horses will go hungry.*

Discuss sustainability of my farm in terms of the harvesting rate h.

Solution: Sustainable for $h < 1$. (There is a positive stable equilibrium.)

That is, if $h < 1$, then I can harvest at this rate forever. (At least according to the model.)

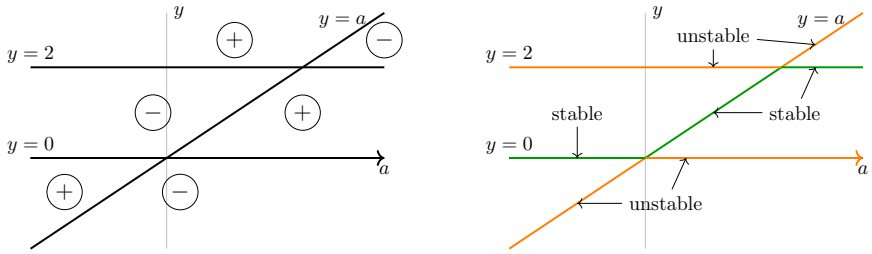
(c) *Identify any bifurcation points.*

Solution: Bifurcation point at $h = 1$ (qualitative change in critical points).

Problem 2. *Consider $y' = y(2-y)(a-y)$, where a is a constant.*

(a) *Draw the bifurcation diagram. (y vs. a)*

Solution: Critical points: $y' = y(2-y)(a-y) = 0 \quad \Rightarrow y = 0,\ y = 2,\ y = a.$



The three lines showing the critical points divide the ay-plane into 6 regions. We mark the regions with ⊕ and ⊖ to indicate where y' is positive or negative. We then translate this into a color coded bifurcation diagram. (We also label the various branches as stable or unstable.)

(b) *For what values of a is this sustainable?*

Solution: Sustainable for $a > 0$.

(c) *Identify all the bifurcation points.*

Solution: Bifurcation points at $a = 0$, $a = 2$.

ES.1803 Differential Equations

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