Problems Day 34, F 3/22/2024
Topic 16: Eigenstuff (day 1)
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Problem 1. (Eigenstuff and linearity) Say A is a 3×3 matrix with eigenvalues $\lambda = 3,\ 4,\ 5$ and corresponding eigenvectors $\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix},\ \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix},\ \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$.

Compute $A \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}, \quad A \left(\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \right), \quad A \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix}$.

Problem 2. Find the eigenvalues and a basis for each eigenspace of the given matrices. Check your answer by multiplying the matrix times the vector.

$$A = \begin{bmatrix} 6 & 5 \\ 1 & 2 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 6 \end{bmatrix}.$$

Problem 3. (Complex λ) $A = \begin{bmatrix} 3 & 5 \\ -5 & 3 \end{bmatrix}$ –same instructions as Problem 2.

Problem 4. (Diagonal matrices) Same instructions as Problem 2.

$$A = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}, \qquad B = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 4 \end{bmatrix}.$$

Problem 5. (Repeated λ) Same instructions as Problem 2.

$$A = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \qquad B = \begin{bmatrix} 2 & 1 \\ 0 & 2 \end{bmatrix} \qquad C = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 2 \\ 0 & 0 & 2 \end{bmatrix}.$$

Why is B called 'defective'?

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ES.1803 Differential Equations

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