Problems Day 39, F 4/5/2024
Topic 20: Step and delta functions
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Problem 1. Solve $x' + 5x = 0$, $x(0) = 1$. Graph the solution.

Problem 2. Solve $x' + 5x = 2\delta(t)$, $x(0^-) = 0$. Graph the solution.

Problem 3. Compute each of the following integrals.

(a) $\displaystyle \int_{-\infty}^{\infty} \delta(t)\, dt.$

(b) $\displaystyle \int_{-5}^{10} \delta(t) + 2\delta(t - 5) + 4\delta(t - 15)\, dt.$

(c) $\displaystyle \int_{-5}^{10} e^{2t} \left[\delta(t) + 2\delta(t - 5) + 4\delta(t - 15) \right]\, dt.$

(d) $\displaystyle \int_{-5}^{15^-} e^{2t} \left[\delta(t) + 2\delta(t - 5) + 4\delta(t - 15) \right]\, dt.$

(e) $\displaystyle \int_{-\infty}^{\infty} e^{\tan t}\delta(t - \pi/4)\, dt$

Problem 4.
(a) Find the general solution to $2x'' + 7x' + 6x = 0$. (Roots $= -3/2,\ -2$.)
(b) Solve $2x'' + 7x' + 6x = 6\delta(t)$, $x(0^-) = 0$, $x'(0^-) = 0$.

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