Problems Day 4, R 2/8/2024

Topic 2: Linear Systems, input response

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Problem 1. Say whether or not the following equations are first-order linear DEs.

(a) $t^2y' + 2ty = 3y^2$

(b) $y' + 7y = \tan t$

(c) $(y' + y)t = 3$

(d) $y' + e^y y = 3t$

Problem 2. Solve each of the following.

(a) $y' + 5y = 0$ (homogeneous).

(b) $y' + 5y = 5, \quad y(0) = 0.$

(c) $y' + 5y = t.$

(d) $y' + 5y = 17 + 9t.$

Problem 3.

(a) State the superposition principle for a first-order linear DE.

(b) Show that the nonlinear DE $y'y = q(t)$ does not satisfy the superposition principle. ($q(t)$ = input, can be any function.)

(c) Show that $y' + p(t)y = q(t)$ does satisfy the superposition principle.

Problem 4. Solve $y' + 5y = \begin{cases} 0 & \text{for } t < 0 \\ 5 & \text{for } 0 < t < 1 \\ t & \text{for } 1 < t \end{cases}$, with $y(0) = 0$, $y(t)$ is continuous.

Problem 5. Consider the family of ellipses $x^2 + 2y^2 = c$. Find an orthogonal family of curves.

ES.1803 Differential Equations

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