Problems Day 45, T 4/16/2024

Topic 22: Fourier series (continued)

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Note: There is a useful integral table on the last page.

Problem 1. Compute the Fourier series of $\text{tri}(t)$, the standard period 2π triangle wave. Do this by computing the integrals for its coefficients.

Problem 2. Let $f(t) = |\sin t|$ (rectified sine curve).

(a) Graph this.

(b) Estimate the decay rate of its Fourier coefficients.

(c) Compute its Fourier series.

(d) Confirm your answer to Part (b).

Problem 3. Say whether each of the following functions is even, odd or neither.

(a) $t^2 \sin(3t)$

(b) $t^2 \sin(3t) + t^2 \cos(3t)$

(c) e^{-t}

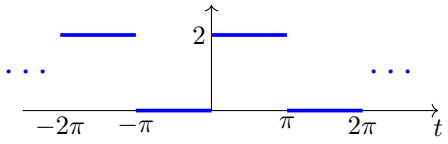
(d) $t \sin(8t)$

(e) $f(t)$ has period 2. $f(t) = e^{-t^2}$ for $0 \le t \le 2$.

(f) $f(t)$ has period 2π; $f(t) = 2\pi t$ for $-\pi < t < \pi$

Problem 4. Let $f(t) = e^{\sin t}$. What is the period of $f(t)$? Estimate the decay rate of its coefficients.

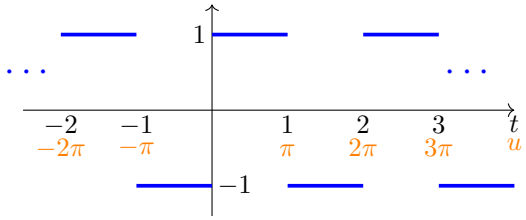
Problem 5. If didn't do this last class: Let $f(t) = 1 + \text{sq}(t)$



Graph of $f(t)$

Find the Fourier series.

Problem 6. If didn't do this last class: Let $g(t)$ have period 2 and $g(t) = \begin{cases} -1 & \text{for } -1 < t < 0 \\ 1 & \text{for } 0 < t < 1. \end{cases}$



Graph of $g(t)$

Find the Fourier series for $g(t)$.

Integrals (for n a positive integer)

1. $\displaystyle\int t\sin(\omega t)\,dt = \frac{-t\cos(\omega t)}{\omega} + \frac{\sin(\omega t)}{\omega^2}.$

1′. $\displaystyle\int_0^{\pi} t\sin(nt)\,dt = \frac{\pi(-1)^{n+1}}{n}.$

2. $\displaystyle\int t\cos(\omega t)\,dt = \frac{t\sin(\omega t)}{\omega} + \frac{\cos(\omega t)}{\omega^2}.$

2′. $\displaystyle\int_0^{\pi} t\cos(nt)\,dt = \begin{cases} \dfrac{-2}{n^2} & \text{for } n \text{ odd} \\ 0 & \text{for } n \neq 0 \text{ even} \end{cases}$

3. $\displaystyle\int t^2\sin(\omega t)\,dt = \frac{-t^2\cos(\omega t)}{\omega} + \frac{2t\sin(\omega t)}{\omega^2} + \frac{2\cos(\omega t)}{\omega^3}.$

3′. $\displaystyle\int_0^{\pi} t^2\sin(nt)\,dt = \begin{cases} \dfrac{\pi^2}{n} - \dfrac{4}{n^3} & \text{for } n \text{ odd} \\ \dfrac{-\pi^2}{n} & \text{for } n \neq 0 \text{ even} \end{cases}$

4. $\displaystyle\int t^2\cos(\omega t)\,dt = \frac{t^2\sin(\omega t)}{\omega} + \frac{2t\cos(\omega t)}{\omega^2} - \frac{2\sin(\omega t)}{\omega^3}.$

4′. $\displaystyle\int_0^{\pi} t^2\cos(nt)\,dt = \frac{2\pi(-1)^n}{n^2}$

If $a \neq b$

5. $\displaystyle\int \cos(at)\cos(bt)\,dt = \frac{1}{2}\left[\frac{\sin((a+b)t)}{a+b} + \frac{\sin((a-b)t)}{a-b}\right]$

6. $\displaystyle\int \sin(at)\sin(bt)\,dt = \frac{1}{2}\left[-\frac{\sin((a+b)t)}{a+b} + \frac{\sin((a-b)t)}{a-b}\right]$

7. $\displaystyle\int \cos(at)\sin(bt)\,dt = \frac{1}{2}\left[-\frac{\cos((a+b)t)}{a+b} + \frac{\cos((a-b)t)}{a-b}\right]$

8. $\displaystyle\int \cos(at)\cos(at)\,dt = \frac{1}{2}\left[\frac{\sin(2at)}{2a} + t\right]$

9. $\displaystyle\int \sin(at)\sin(at)\,dt = \frac{1}{2}\left[-\frac{\sin(2at)}{2a} + t\right]$

10. $\displaystyle\int \sin(at)\cos(at)\,dt = -\frac{\cos(2at)}{4a}$

ES.1803 Differential Equations

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